WIENER-HOPF ANALYSIS OF THE AXIAL SYMMETRIC WAVE DIFFRACTION PROBLEM FOR A CIRCULAR WAVEGUIDE CAVITY

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Abstract

The axial symmetric diffraction problem for a circular waveguide cavity is rigorously analyzed by means of the Wiener-Hopf technique. The incident field is taken as a uniform ring source located on the interior cylindrical surface of the cavity. Introducing the Fourier transform of the scattered field and applying boundary conditions in the transform domain, the problem is formulated in the terms of the Wiener-Hopf equation. The Wiener-Hopf equation is solved exactly via the factorization procedure leading to the formal solution. An approximate solution of such equation is derived. The scattered field inside and outside of the cavity is evaluated and the partial case is considered.

1. Introduction

The analysis of electromagnetic wave scattering by open-ended metallic waveguide cavities is an important subject in radar cross section (RSC) reduction and target identification studies [1, 2] since these obstacles contribute significantly to the RSC due to the interior irradiation. This problem serves as a simple model of duct structures such as jet engine intakes of aircrafts and cracks occurring on surfaces of general complicated bodies. There have been a number of investigations on the scattering by two-dimensional (2-D) and three-dimensional (3-D) cavities of various shapes based on high-frequency and numerical techniques [3-9]. The solutions obtained by the former and latter approaches, however, are not valid at low - and high - frequency limits, respectively.

The Wiener -Hopf technique [10-12] is known as powerful tool for analyzing electromagnetic wave problems associated with canonical geometries, which is mathematically rigorous in the sense that the edge condition is explicitly incorporated into the analysis. In [13], we have considered a finite parallel - plate waveguide with a planar termination at the open end as an example of simple 2-D cavity structures, and solved the plane wave diffraction problem rigorously using the Wiener-Hopf technique. As a result, an efficient approximate solution has been obtained, which is valid for cavity depth greater than the incident wavelength. We have further considered 2-D material - loaded cavities formed by finite and semi - infinite parallel - plate waveguides, and carried out a rigorous RSC analysis by means of the Wiener -Hopf technique [14-16]. It has been shown by numerical computation that our results are valid over a broad frequency range and can be used as a reference solution for validating more general - purpose computer codes based on approximate methods. Most of our Wiener -Hopf results related to parallel - plate waveguide cavities are summarized in detail in [17].

In this paper, we shall consider a 3-D cavity formed by a finite circular waveguide with a planar termination at the open end, and analyze the axial symmetric diffraction problem by means of the Wiener-Hopf technique. This cavity geometry provides a more realistic model of jet engine intakes of aircrafts, and is important in the RSC studies. The incident field is assumed to be a uniform ring source located on the interior cylindrical surface of the cavity. This assumption simplifies the analysis procedure since the problem is then reduced to a scalar case. The method of solution is similar to that we have developed for the analysis of the parallel-plate waveguide cavity, but is more complicated because of the cylindrical geometry [18-20].

In Section 2, the Fourier transform for the unknown scattered field is introduced and transformed wave equations are derived by taking the Fourier transform of the Helmholtz equation. It is to be noted that the transformed wave equations involve unknown inhomogeneous terms occuring due to a medium discontinuity. In Section 3, these transformed wave equations are solved by expanding the inhomogeneous terms into the Fourier - Bessel series, and the scattering field in the transform domain is derived. In Section 4, the problem is formulated in terms of the Wiener - Hopf equation satisfied by unknown spectral functions, where the unknown Fourier - Bessel coefficients are involved. In Section 5, the Wiener - Hopf equation is solved exactly via the factorization and decomposition procedure leading to the formal solution, which involves infinite series with unknown coefficients and infinite brunch - cut integrals with unknown integrands. In Section 6, we shall develop approximate methods for determining the unknowns and evaluating the brunch - cut integrals based on rigorous asymptotics and derive the approximate solution to the Wiener - Hopf equation. Section 7 discusses the derivation of the scattered field inside and outside the cavity by taking the Fourier inverse of the solution in the transform domain. In Section 8, the partial case as wave diffraction by the semi infinite cylinder with internal plate termination is considered. Some concluding remarks are given in Section 9.

The time factor is assumed to be $e^{-i\omega t}$ and suppressed throughout this paper.

2. Transformed wave equations

We consider a 3-D cavity formed by a finite circular waveguide with a planar termination at the open end, as shown in Fig. 1, where the cavity surface is perfectly conducting and of zero thickness, being defined by

$$S=\left\{(\rho,\varphi,z)|\rho=b,\ 0\le\varphi\le 2\pi, |z|\le L\right\}\cup\left\{(\rho,\varphi,z)|\, 0\le\rho\le b, 0\le\varphi\le 2\pi,\ z=-L\right\}. \quad (1)$$

Here (ρ,φ,z) are cylindrical coordinates. The cavity is assumed to be excited by a hypothetical generator with voltage of unit amplitude across an infinitesimally small gap at $z=d\,(<|L|)$. Thus the applied electric field becomes a uniform ring source and is given by [11]

$$e_z^i(z)=\delta(z-d), \quad (2)$$

where $\delta(\cdot)$ is a delta function. This ring source is located at $(\rho,z)=(b-0,d)$ and excites TM waves along the cylindrical surface of the cavity.

In view of the axial symmetry of the problem, nonzero components of the scattered electromagnetic field are derived through the relation

$$(e_z, e_\rho, h_\varphi) = \left[-\frac{1}{i\omega\varepsilon}\left(\frac{\partial^2 \phi}{\partial z^2} + k^2 \phi\right),\ -\frac{1}{i\omega\varepsilon}\frac{\partial^2 \phi}{\partial \rho\, \partial z},\ -\frac{\partial \phi}{\partial \rho} \right] \tag{3}$$

with $k = \omega(\varepsilon\ \mu\ )^{1/2}$ being the free-space wave number, where $\phi\,(\rho, z)$ satisfies the scalar Helmholtz equation. For analytical convenience, we assume that the medium is slightly lossy as in $k = k_1 + i\,k_2$ with $0 < k_2 << k_1$. The solution for real k is obtained by letting $k_2 \to +0$ at the end of analysis.

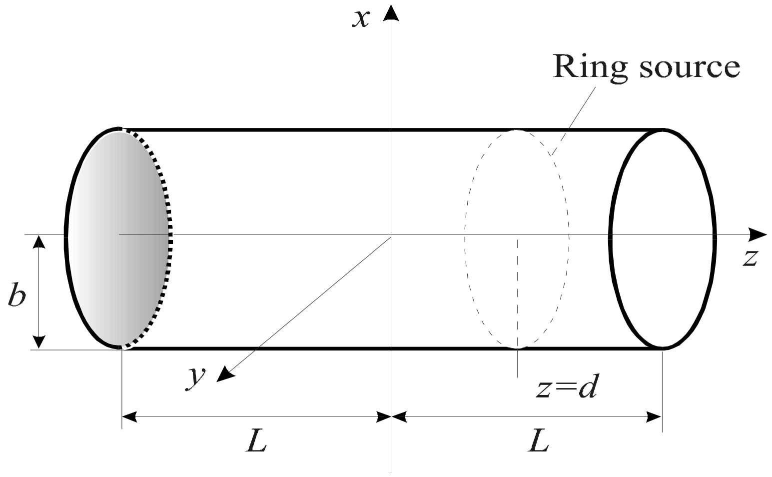

Fig. 1. Geometry of the problem.

Let the total field $\phi^{\,t}(\rho, z)$ be given by

$$\phi^{\,t}(\rho, z) = \begin{cases} \phi^{\,i}(\rho, z) + \phi\,(\rho, z) & \text{for } 0 < \rho < b, \\ \phi\,(\rho, z) & \text{for } \rho > b, \end{cases} \tag{4}$$

where $\phi^{\,i}(\rho, z)$ is the field excited in an infinitely long circular waveguide due to the ring source as given by (2), which takes the form

$$\phi^{\,i}(\rho, z) = \frac{\omega\varepsilon}{2\pi i} \int_{-\infty}^{+\infty} \frac{I_0(\gamma_\beta \rho)}{\gamma_\beta^2\, I_0(\gamma_\beta b)} e^{-i\beta(z-d)} d\beta. \tag{5}$$

In (5), $\gamma_\beta = (\beta^2 - k^2)^{1/2}$ with $\mathrm{Re}\,\gamma_\beta > 0$, and $I_0(\cdot)$ is the modified Bessel function of the first kind.

Our problem is to determine the field scattered by the cavity due to the incidence of $\phi^{\,i}(\rho, z)$. Let us define the Fourier transform of the scattered field $\phi\,(\rho, z)$ in (4) with respect to z as

$$\Phi(\rho, \alpha) = (2\pi)^{-1/2} \int_{-\infty}^{+\infty} \phi\,(\rho, z) e^{i\alpha z}\, dz, \tag{6a}$$

where $\alpha = \mathrm{Re}\,\alpha + i\,\mathrm{Im}\,\alpha\ (\equiv \sigma + i\tau)$ .In view of the radiation condition, it follows that

$\phi(\rho,z)=O(e^{-k_2|z|})$ as $|z|\to\infty$. Hence we see that $\Phi(\rho,\alpha)$ is regular in the strip $|\tau|<k_2$ of the complex α-plane. Introducing the Fourier integrals as

$$\Phi_\pm(\rho,\alpha)=\pm\frac{1}{\sqrt{2\pi}}\int_{\pm L}^{\pm\infty}\phi(\rho,z)\,e^{i\alpha(z\mp L)}dz\,, \tag{6b}$$

$$\Phi_1(\rho,\alpha)=\frac{1}{\sqrt{2\pi}}\int_{-L}^{+L}\phi^{\,t}(\rho,z)e^{i\alpha z}dz\,, \tag{6c}$$

it is found that $\Phi_\pm(\rho,\alpha)$ are regular in the half-planes $\tau \gtrless \mp k_2$ and $\Phi_1(\rho,\alpha)$ is an entire function. Using the notation as given by (6b,c), we may express $\Phi(\rho,\alpha)$ as

$$\Phi(\rho,\alpha)=\Psi(\rho,\alpha)+\Phi_1(\rho,\alpha)-\Phi^{i}(\rho,\alpha) \tag{7}$$

for $0<\rho<b$, where

$$\Phi^{i}(\rho,\alpha)=\frac{\omega\,\varepsilon}{(2\pi)^{1/2}\,i}\,\frac{I_0(\gamma\rho)\,e^{i\alpha d}}{\gamma^2 I_0(\gamma b)}\,, \tag{8}$$

$$\Psi(r,\alpha)=e^{-i\alpha L}\Psi_-(r,\alpha)+e^{+i\alpha L}\Psi_+(r,\alpha) \tag{9}$$

with $\gamma=(\alpha^2-k^2)^{1/2}$ for $\mathrm{Re}\,\gamma>0$, and

$$\Psi_-(\rho,\alpha)=\Phi_-(\rho,\alpha)+Q_-(\rho,\alpha),\;\;\Psi_+(\rho,\alpha)=\Phi_+(\rho,\alpha)+Q_+(\rho,\alpha), \tag{10}$$

$$Q_\pm(\rho,\alpha)=\pm\frac{\omega\,\varepsilon}{(2\pi)^{3/2}}\int_{-\infty+i\varepsilon_\pm}^{+\infty+i\varepsilon_\pm}\frac{I_0(\gamma_\beta\rho)e^{i\beta(d\mp L)}}{\gamma_\beta^2 I_0(\gamma_\beta b)}\,\frac{d\beta}{\alpha-\beta}\,. \tag{11}$$

In (11), the constants $\varepsilon_\pm$ are taken such that $-k_2<\varepsilon_+<\tau$ and $\tau<\varepsilon_-<k_2$.

We take the Fourier transform of the Helmholtz equation and use the boundary condition at the endplate of the cavity as well as the radiation condition. Applying the method established in our previous papers [13, 19] we derive the transformed wave equations as follows

$$\hat{T}\Phi(\rho,\alpha)=0 \quad \text{in} \quad \rho>b \quad \text{for} \quad |\tau|<k_2\,, \tag{12a}$$

$$\hat{T}\,\Psi_-(\rho,\alpha)=\alpha\,f(\rho) \quad \text{in} \quad 0<\rho<b \quad \text{for} \quad \tau<k_2\,, \tag{12b}$$

$$\hat{T}\left[\Phi_1(\rho,\alpha)+e^{i\alpha L}\Psi_+(\rho,\alpha)\right]=-\alpha\,e^{-i\alpha L}g(\rho) \quad \text{in} \quad 0<\rho<b \quad \text{for} \quad \tau>-k_2\,, \tag{12c}$$

where

$$\hat{T}=\left[d^2/d\rho^2+\rho^{-1}d/d\rho-\gamma^2\right], \tag{13}$$

and $f(\rho)$ and $g(\rho)$ are unknown inhomogeneous terms defined by

$$f(\rho)=i(2\pi)^{-1/2}\phi^{\,t}(\rho,-L-0),\;\; g(\rho)=i(2\pi)^{-1/2}\phi^{\,t}(\rho,-L+0)\,. \tag{14}$$

3. Field representation in the transform domain

Since the scattered field for the region $\rho > b$ must vanish as $\rho \to \infty$ according to the radiation condition, we find that the solution of (12a) is expressed as

$$\Phi(\rho,\alpha) = \Psi(b,\alpha)\frac{K_0(\gamma\rho)}{K_0(\gamma b)} \quad \text{for} \ \ \tau < |\,k_2\,| \tag{15}$$

with $K_0(\cdot)$ being the modified Bessel function of the second kind, where the boundary condition at $\rho = b$ has been taken into account. The derivation of a field representation for the region $0 < \rho < b$ is complicated since the transformed wave equations (12b,c) contain the unknown inhomogeneous terms $f(\rho)$ and $g(\rho)$ as given by (14). We now expand $f(\rho)$ and $g(\rho)$ into the convergent Fourier-Bessel series, as in

$$f(\rho) = \sum_{n=1}^{\infty} f_n J_0(\xi_n\,\rho/b), \qquad g(\rho) = \sum_{n=1}^{\infty} g_n J_0(\xi_n\,\rho/b) \tag{16}$$

for $0 < \rho < b$ with $J_0(\cdot)$ being the Bessel function, where f_n and g_n for $n = 1,2,3,\ldots$ are unknown coefficients and ξ_n for $n = 1,2,3,\ldots$ denotes the zeros of $J_0(\cdot)$. Let us substitute (16) into (12b,c) and use the finite Fourier-Bessel integral transformation

$$R(\rho,\alpha) = \sum_{n=1}^{\infty} a_{\xi_n}(\alpha) Z_0(\xi_n\,\rho/b), \quad a_{\xi_n}(\alpha) = \int_0^b R(\rho,\alpha) Z_0(\xi_n\,\rho/b)\rho\, d\rho \tag{17}$$

with $Z_0(\xi_n\,\rho/b) = \sqrt{2}[bJ_1(\xi_n)]^{-1} J_0(\xi_n\rho/b)$, where $R(\rho,\alpha)$ is any function that allows the Fourier-Bessel series expansion, and $a_{\xi_n}(\alpha)$ for $n = 1,2,3,\ldots$ are the expansion coefficients for obtaining the particular solution of (12b,c). Then by following a procedure similar to that developed in [13] and using the boundary condition at $\rho = b$, we arrive at the solution of (12b,c) with the result that

$$\Psi_-(\rho,\alpha) = \Psi_-(b,\alpha)\frac{I_0(\gamma\,\rho)}{I_0(\gamma b)} - \sum_{n=1}^{\infty} \frac{\alpha f_n}{\alpha^2 + \gamma_n^2} I_0(-i\xi_n\rho/b), \tag{18a}$$

$$\begin{aligned} &\Phi_1(\rho,\alpha) + \Psi_+(\rho,\alpha)e^{i\alpha L} = \\ &\qquad = \Psi_+(b,\alpha)e^{i\alpha L}\frac{I_0(\gamma\,\rho)}{I_0(\gamma b)} + \sum_{n=1}^{\infty} \frac{\alpha e^{-i\alpha L} g_n}{\alpha^2 + \gamma_n^2} I_0(-i\xi_n\rho/b), \end{aligned} \tag{18b}$$

where $\gamma_n = [(\xi_n/b)^2 - k^2]^{1/2}$.

Summarizing the above results, we obtain that

$$\Phi(\rho,\alpha) = \begin{cases} \Psi(b,\alpha)\dfrac{K_0(\gamma\,\rho)}{K_0(\gamma\,b)} \quad \text{for} \ \rho > b, \\ \Psi(b,\alpha)\dfrac{I_0(\gamma\,\rho)}{I_0(\gamma\,b)} - \displaystyle\sum_{n=1}^{\infty} \frac{\alpha e^{-i\alpha L}(f_n - g_n)}{\alpha^2 + \gamma_n^2} I_0(-i\xi_n\,\rho/b) - \Phi^i(\rho,\alpha) \\ \qquad\qquad \text{for} \ 0 < \rho < b. \end{cases} \tag{19}$$

Equation (19) gives the scattered field representation in the Fourier transform domain and holds in the strip $|\tau| < k_2$.

4. Wiener-Hopf equation

Using (19) and taking into account (3), we find that the Fourier transform of the electric component $e_z(b,z)$ can be written as

$$E(b,\alpha) = e^{-i\alpha L}E_-(b,\alpha) + e^{+i\alpha L}E_+(b,\alpha) = \frac{\gamma^2}{i\omega\varepsilon}\Psi(b,\alpha), \tag{20a}$$

where

$$E_\pm(b,\alpha) = \left(\gamma^2/i\omega\varepsilon\right)\Psi_\pm(b,\alpha). \tag{20b}$$

Here, $E_-(b,\alpha)$ and $E_+(b,\alpha)$ are the Fourier integrals of the electric component $e_z(b,z)$ over the intervals $-\infty < z < -L$ and $L < z < \infty$, respectively. We differentiate (19) with respect to ρ and set $\rho = b \pm 0$ in the resultant equations. Making use of (3), it follows that

$$H(b+0,\alpha) = \Psi(b,\alpha)\frac{\gamma K_1(\gamma b)}{K_0(\gamma b)} \quad , \tag{21a}$$

$$\begin{aligned} H(b-0,\alpha) = -\Psi(b,\alpha)\frac{\gamma I_1(\gamma b)}{I_0(\gamma b)} - \frac{i\alpha}{b}\sum_{n=1}^{\infty}\frac{e^{-i\alpha L}\xi_n(f_n-g_n)}{\alpha^2+\gamma_n^2}I_1(-i\xi_n) - \\ - H^i(b,\alpha) \end{aligned} \tag{21b}$$

with $H(b\pm 0,\alpha)$ being the Fourier transform of the magnetic field components $h_\varphi(b\pm 0,z)$, where

$$H^i(b,\alpha) = -\frac{\omega\varepsilon}{(2\pi)^{1/2}i}\,\frac{e^{i\alpha d}\,I_1(\gamma b)}{\gamma I_0(\gamma b)}. \tag{22}$$

Taking the difference of (21a) and (21b) and making some arrangements, we derive that

$$\begin{aligned} J_1(b,\alpha) = \frac{i\omega\varepsilon}{b}\frac{e^{-i\alpha L}E_-(b,\alpha) + e^{i\alpha L}E_+(b,\alpha)}{M(\alpha)} + \\ + \frac{i\alpha}{b}\sum_{n=1}^{\infty}\frac{e^{-i\alpha L}\xi_n(f_n-g_n)}{\alpha^2+\gamma_n^2}I_1(-i\xi_n) + U(b,\alpha), \end{aligned} \tag{23}$$

where $J_1(b,\alpha)$ denotes the Fourier integral of the unknown current on the cylindrical surface of the cavity over the interval $-L < z < L$, and

$$U(b,\alpha) = H^i(b,\alpha) - J_+(b,\alpha) - J_-(b,\alpha), \tag{24}$$

$$M(\alpha) = \gamma^2 K_0(\gamma b) I_0(\gamma b) \tag{25}$$

with

$$J_\pm(b,\alpha) = \mp \frac{\omega\,\varepsilon\, e^{\pm i\alpha L}}{(2\pi)^{3/2}} \int_{-\infty+i\varepsilon_\pm}^{+\infty+i\varepsilon_\pm} \frac{I_1(\gamma_\beta b)e^{i\beta(d\mp L)}}{\gamma_\beta I_0(\gamma_\beta b)} \frac{d\beta}{\alpha-\beta}. \tag{26}$$

Equation (23) is the Wiener-Hopf equation satisfied by the unknown functions $E_-(b,\alpha)$, $E_+(b,\alpha)$ and $J_1(b,\alpha)$, where the unknown Fourier-Bessel coefficients f_n and g_n are also involved. In order to arrange (23) into a convenient form, it is desirable to investigate the relationship between the unknown functions and the unknown coefficients.

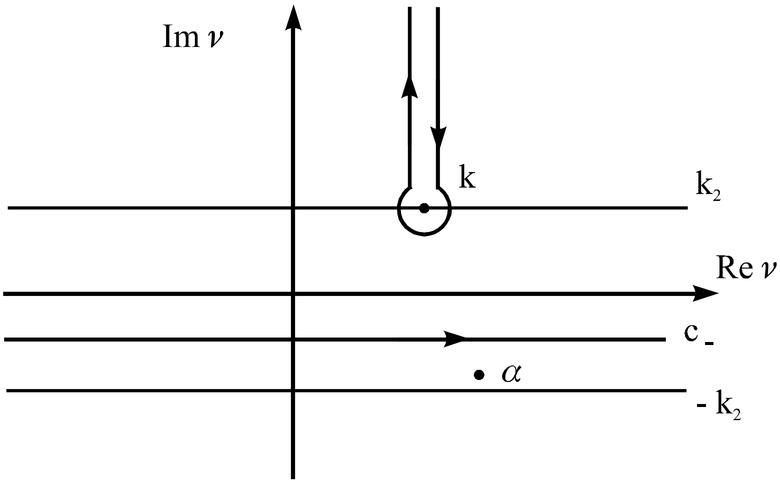

Fig. 2. Integration path for evaluation of the integral in (35).

As has already been shown, $\Psi_-(\rho,\alpha)$ and $\Psi_+(\rho,\alpha)$ are regular in the half-planes $\tau < k_2$ and $\tau > -k_2$, respectively, while $\Phi_1(\rho,\alpha)$ is an entire function. Therefore it follows that

$$\lim_{\alpha\to -i\gamma_m} (\alpha + i\gamma_m)\Psi_-(\rho,\alpha) = 0, \tag{27}$$

$$\lim_{\alpha\to i\gamma_m} (\alpha - i\gamma_m)[\Phi_1(\rho,\alpha) + \Psi_+(\rho,\alpha)e^{i\alpha L}] = 0 \tag{28}$$

for $m = 1,2,3,\ldots$. Substituting (18a) and (18b) into (27) and (28), respectively and taking into account (20b), we derive, after some manipulations, that

$$f_m - g_m = -\frac{2i\omega\,\varepsilon}{\gamma_m\,\xi_m\, I_1(-i\xi_m)}[E_-(b,-i\gamma_m) - e^{-2\gamma_m L}E_+(b,+i\gamma_m)]. \tag{29}$$

Substituting (29) into (23) gives

$$\begin{aligned} J_1(b,\alpha) &= \frac{i\omega\,\varepsilon}{b}\frac{e^{-i\alpha L}E_-(b,\alpha) + e^{i\alpha L}E_+(b,\alpha)}{M(\alpha)} + \\ &+ \frac{2\omega\varepsilon\alpha}{b}\sum_{n=1}^{\infty} \frac{e^{-i\alpha L}[E_-(b,-i\gamma_n) - e^{-2\gamma_n L}E_+(b,+i\gamma_n)]}{\gamma_n(\alpha^2+\gamma_n^2)} + U(b,\alpha) \end{aligned} \tag{30}$$

for $|\tau| < k_2$. Equation (30) is the desired Wiener-Hopf equation.

5. Exact solution

The kernel function $M(\alpha)$ is factorized as [11]

$$M(\alpha)=M_{+}(\alpha)M_{-}(\alpha), \tag{31}$$

where

$$\begin{aligned} M_{+}(\alpha)=i\left[\frac{i\pi}{2}J_0(kb)H_0^1(kb)\right]^{1/2}(\alpha+k)\exp\left[-\frac{i\,kb}{2}+\frac{i\,k\gamma}{\pi}\ln\left(\frac{\alpha-\gamma}{k}\right)+q(\alpha)\right]\times \\ \times\exp\left[\frac{ib\alpha}{\pi}\left(1-C+\ln\frac{2\pi}{kb}+i\frac{\pi}{2}\right)\right]\prod_{n=1}^{\infty}\left(1+\frac{\alpha}{i\gamma_n}\right)e^{i\alpha b/n\pi} \end{aligned} \tag{32}$$

with $H_0^{(1)}(\cdot)$ being the Hankel function of the first kind, and

$$q(\alpha)=\frac{b}{\pi}\int_0^{\infty}\left[1-\frac{2}{\pi t b}\frac{1}{J_0^2(tb)+Y_0^2(tb)}\right]\ln\left[1+\frac{\alpha}{(k^2-t^2)^{1/2}}\right]dt\,. \tag{33}$$

In the above, $Y_0(\cdot)$ is the Neumann function, and $C=0.57721566\ldots$ is Euler's constant. Here $M_{-}(\alpha)=M_{+}(-\alpha)$ and split functions $M_{\pm}(\alpha)$ are regular and nonzero in $\tau\gtrless\mp k_2$, and show the asymptotic behavior $M_{\pm}(\alpha)=O(\alpha^{1/2})$ as $\alpha\to\infty$ with $\tau\gtrless\mp k_2$.

Using the edge condition, it is found that the unknown functions in (30) behave like

$$E_{+}(b,\alpha)=O(\alpha^{-1/2})\ \text{ for }\ \tau>-k_2\,,\ E_{-}(b,\alpha)=O(\alpha^{-2/3})\ \text{ for }\ \tau<k_2\,, \tag{34a}$$

$$J_1(b,\alpha)e^{i\alpha L}=O(\alpha^{-5/3})\ \text{ for }\ \tau>-k_2\,,\ J_1(b,\alpha)e^{-i\alpha L}=O(\alpha^{-3/2})\ \text{ for }\ \tau<k_2 \tag{34b}$$

as $\alpha\to\infty$. Multiplying both sides of (30) by $e^{i\alpha L}M_{+}(\alpha)$ and applying the decomposition procedure with the aid of (34a,b), we derive that

$$\begin{aligned} \frac{E_{-}(b,\alpha)}{M_{-}(\alpha)}-\frac{1}{2\pi i}\int_{-\infty+ic_-}^{+\infty+ic_-}\frac{e^{2i\nu L}E_{+}(b,\nu)}{M_{-}(\nu)(\nu-\alpha)}d\nu+ \\ +\sum_{n=1}^{\infty}\frac{M_{+}(i\gamma_n)\,[E_{-}(b,-i\gamma_n)-e^{-2\gamma_n L}E_{+}(b,+i\gamma_n)]}{i\gamma_n(\alpha-i\gamma_n)}=R_{-}(\alpha), \end{aligned} \tag{35}$$

where

$$\begin{aligned} R_{-}(\alpha)=\frac{b}{(2\pi)^{5/2}}\int_{-\infty+ic_-}^{+\infty+ic_-}\frac{M_{+}(\nu)}{\nu-\alpha}\left(\int_{-\infty+i\varepsilon_-}^{+\infty+i\varepsilon_-}\frac{I_1(\gamma_\beta b)e^{i\beta(d+L)}}{\gamma_\beta I_0(\gamma_\beta b)}\frac{d\beta}{\nu-\beta}-\right. \\ \left.-2\pi i\frac{e^{i\nu(d+L)}I_1(\gamma_\nu b)}{\gamma_\nu I_0(\gamma_\nu b)}\right)d\nu \end{aligned} \tag{36}$$

with $\tau<\varepsilon_-<c_-<k_2$. It is seen from (20b), (25) (32) and (33) that singularities associated with the integral in (35) for $\operatorname{Im}\nu>c_-$ are simple poles at $\nu=i\gamma_m$ with $m=1,2,3,\ldots$, and a branch point at $\nu=k$. We now choose a branch cut emanating from $\nu = k$ as a straight line that is parallel to the imaginary axis and goes to infinity in the upper half-plane. Then evaluating the integral by enclosing the contour into the upper half-plane (see Fig. 2), we derive, after some manipulations, that

$$E_-(b,\alpha) + M_-(\alpha)\left[J_E^{(1)}(\alpha) + \sum_{n=1}^{\infty} \frac{M_+(i\gamma_n)E_-(b,-i\gamma_n)}{i\gamma_n(\alpha - i\gamma_n)} \right] = M_-(\alpha)R_-(\alpha), \quad (37)$$

where

$$J_E^{(1)}(\alpha) = \frac{1}{2}\int_k^{+i\infty+k} \frac{e^{2i\nu L}M_+(\nu)E_+(b,\nu)}{\gamma_\nu^2 K_0(\gamma_\nu b)\left[K_0(\gamma_\nu b) - i\pi I_0(\gamma_\nu b)\right]} \frac{d\nu}{\nu - \alpha} \quad (38)$$

with $\gamma_\nu = (\nu^2 - k^2)^{1/2}$ for $\mathrm{Re}\gamma_\nu > 0$. In (38), the contour is the one running parallel to the imaginary axis on the right-hand side of the branch cut. Next, multiplying both sides of (30) by $e^{-i\alpha L}M_-(\alpha)$ and following a procedure similar to that employed to obtain (37) yields,

$$E_+(b,\alpha) - M_+(\alpha)\left[J_E^{(2)}(\alpha) + \sum_{n=1}^{\infty} \frac{e^{-4\gamma_n L}M_+(i\gamma_n)E_+(b,i\gamma_n)}{i\gamma_n(\alpha + i\gamma_n)} \right] = M_+(\alpha)R_+(\alpha), \quad (39)$$

where

$$\begin{aligned} R_+(\alpha) = \frac{b}{(2\pi)^{5/2}} \int_{-\infty+ic_+}^{+\infty+ic_+} \frac{M_-(\nu)}{\nu - \alpha} & \left(\int_{-\infty+i\varepsilon_+}^{+\infty+i\varepsilon_+} \frac{I_1(\gamma_\beta b)e^{i\beta(d-L)}}{\gamma_\beta I_0(\gamma_\beta b)} \frac{d\beta}{\nu - \beta} - \right. \\ & \left. - 2\pi i \frac{e^{i\nu(d-L)}I_1(\gamma_\nu b)}{\gamma_\nu I_0(\gamma_\nu b)} \right) d\nu, \end{aligned} \quad (40)$$

$$J_E^{(2)}(\alpha) = \frac{1}{2}\int_{-k}^{-i\infty-k} \frac{e^{-2i\nu L}M_-(\nu)E_-(b,\nu)}{\gamma_\nu^2 K_0(\gamma_\nu b)\left[K_0(\gamma_\nu b) - i\pi I_0(\gamma_\nu b)\right]} \frac{d\nu}{\nu - \alpha}. \quad (41)$$

The integrands in (38) and (41) decay exponentially along the paths of integration. Taking into account the singularities and using the residue theorem, $R_\pm(\alpha)$ defined by (36) and (40) are evaluated with the result that

$$R_\pm(\alpha) = \mp(2\pi)^{-1/2} \sum_{m=1}^{\infty} \frac{M_+(i\gamma_m)e^{\pm\gamma_m(d\mp L)}}{i\gamma_m(\alpha \pm i\gamma_m)}. \quad (42)$$

Now (37) and (39) give the exact solution to the Wiener-Hopf equation (30). These equations are formal since infinite series with unknown coefficients and infinite branch-cut integrals with unknown integrands are involved.

6. Approximate solution

First we shall consider the function $J_E^{(1)}(\alpha)$ defined by (38) and derive its approximate expression convenient for numerical computation. For this purpose, we exchange the variable of an integration in (38) as follows $\nu = it + k$ with the result that

$$J_E^{(1)}(\alpha)=\frac{b^2e^{2ikL}}{2}\int_0^\infty \frac{e^{-2tL}M_+(it+k)E_+(b,it+k)}{R(y)}\frac{dt}{t-i(k-\alpha)}, \tag{43}$$

where $y=it(it+2k)b^2$ and

$$R(y)=yK_0\left(\sqrt{y}\right)\left[K_0\left(\sqrt{y}\right)-i\pi I_0\left(\sqrt{y}\right)\right]. \tag{44}$$

Here

$$|\alpha-k|>0 \text{ and } -\pi/2<\arg(\alpha-k)<3\pi/2. \tag{45}$$

Let us apply the Taylor's theorem for evaluating (44) for $t\to 0$ as follows

$$R(y)=R_0(t)+O(t^{3/2}(\ln t)^2), \tag{46}$$

where

$$R_0(t)=2itkb^2K_0\left(i^{1/2}\sqrt{2ktb^2}\right)\left[K_0\left(i^{1/2}\sqrt{2ktb^2}\right)-i\pi I_0\left(i^{1/2}\sqrt{2ktb^2}\right)\right]=O(t(\ln t)^2). \tag{47}$$

Then taking into account the exponentially decaying of the integrand (43) we can express $J_E^{(1)}(\alpha)$ for large $|k|L$ keeping only the leading term for the asymptotic expansion with the result that

$$J_E^{(1)}(\alpha)\sim\frac{1}{2}e^{2ikL}b^2\chi(\alpha)M_+(k)E_+(b,k), \tag{48}$$

and

$$\chi(\alpha)=\int_0^\infty\frac{e^{-2tL}}{[t-i(k-\alpha)]R_0(t)}dt. \tag{49}$$

The integral (49) is uniformly convergent because of the integrated singularity (47) and conditions (45).

A similar procedure can also be applied for asymptotic evaluation of $J_E^{(2)}(\alpha)$ defined by (41), and we derive that

$$J_E^{(2)}(\alpha)\sim\frac{1}{2}e^{2ikL}b^2\chi(-\alpha)M_+(k)E_-(b,-k). \tag{50}$$

It follows from the (32), (34a) that

$$M_+(i\gamma_n)\sim(\gamma_n)^{1/2}, \tag{51}$$

$$E_-(b,-i\gamma_n)\sim C_1(b\gamma_n)^{-2/3},\quad E_+(b,i\gamma_n)\sim C_2(b\gamma_n)^{-1/2} \tag{52}$$

as $n\to\infty$, where C_1 and C_2 are unknown constants which are independent of n. Therefore, the infinite series contained in (37) and (39) are approximated with a choice of a large positive N by

$$\sum_{n=1}^{\infty}\frac{M_{+}(i\gamma_n)E_{-}(b,-i\gamma_n)}{i\gamma_n(\alpha-i\gamma_n)}\approx\sum_{n=1}^{N}\frac{M_{+}(i\gamma_n)E_{-}(b,-i\gamma_n)}{i\gamma_n(\alpha-i\gamma_n)}+C_1S_N^{(1)}(\alpha),\tag{53a}$$

$$\sum_{n=1}^{\infty}\frac{e^{-4\gamma_n L}M_{+}(i\gamma_n)E_{+}(b,i\gamma_n)}{i\gamma_n(\alpha+i\gamma_n)}\approx\sum_{n=1}^{N}\frac{e^{-4\gamma_n L}M_{+}(i\gamma_n)E_{+}(b,i\gamma_n)}{i\gamma_n(\alpha+i\gamma_n)}+C_2S_N^{(2)}(\alpha),\tag{53b}$$

where

$$S_N^{(1)}(\alpha)=\sum_{n=N+1}^{\infty}\frac{b^{1/2}(b\gamma_n)^{-7/6}}{i(\alpha-i\gamma_n)},\tag{54a}$$

$$S_N^{(2)}(\alpha)=\sum_{n=N+1}^{\infty}\frac{e^{-4\gamma_n L}b^{1/2}(b\gamma_n)^{-1}}{i(\alpha+i\gamma_n)}.\tag{54b}$$

Using (48), (50), (53a-b) and (54) in formal solution as given by (37) and (39), we derive the approximate expressions of $E_{-}(\alpha)$ and $E_{+}(\alpha)$ as follows

$$\begin{aligned}E_{-}(b,\alpha)\approx{}&M_{-}(\alpha)R_{-}(\alpha)-\frac{1}{2}e^{2ikL}b^2\chi(\alpha)M_{-}(\alpha)M_{+}(k)E_{+}(b,k)\\&-M_{-}(\alpha)\left[\sum_{n=1}^{N}\frac{M_{+}(i\gamma_n)E_{-}(b,-i\gamma_n)}{i\gamma_n(\alpha-i\gamma_n)}+C_1S_N^{(1)}(\alpha)\right],\end{aligned}\tag{55a}$$

$$\begin{aligned}E_{+}(b,\alpha)\approx{}&M_{+}(\alpha)R_{+}(\alpha)+\frac{1}{2}e^{2ikL}b^2\chi(-\alpha)M_{+}(\alpha)M_{+}(k)E_{-}(b,-k)+\\&+M_{+}(\alpha)\left[\sum_{n=1}^{N}\frac{e^{-4\gamma_n L}M_{+}(i\gamma_n)E_{+}(b,i\gamma_n)}{i\gamma_n(\alpha+i\gamma_n)}+C_2S_N^{(2)}(\alpha)\right].\end{aligned}\tag{55b}$$

Equations (55a,b) give the approximate solution of the Wiener - Hopf equation (30), and are valid for large N and $|k|L$, where unknowns $E_{+}(b,k)$, $E_{-}(b,-k)$, $E_{+}(b,i\gamma_n)$, $E_{-}(b,-i\gamma_n)$ for $n=1,2,3,\ldots,N$, and C_1, C_2 are contained. In order to determine these unknowns, we set $\alpha=-k$, $\alpha=-i\gamma_m$ and $\alpha=k$, $\alpha=i\gamma_m$ for $m=1,2,3,\ldots,N+1$ in (55a) and (55b) respectively. These procedures lead to the two sets of $2N+2$ equations, where $E_{-}(b,-i\gamma_{N+1})$ and $E_{+}(b,i\gamma_{N+1})$ are involved. Since N is a large positive integer, we can employ (52) to replace $E_{-}(b,-i\gamma_{N+1})$ and $E_{+}(b,i\gamma_{N+1})$ by their asymptotic behavior containing C_1 and C_2. Thus, the two sets of $(2N+2)\times(2N+2)$ matrix equations are derived. These equations can be solved numerically with high accuracy.

7. Scattered field representation

The scattered field in the real space is obtained by taking the Fourier inverse of (19) according to the formula

$$\phi(\rho,z)=(2\pi)^{-1/2}\int_{-\infty+ic}^{+\infty+ic}\Phi(\rho,\alpha)e^{-i\alpha z}d\alpha,\tag{56}$$

where $-k_2 < c < k_2$. First we shall consider the field inside the cavity. Taking into account the second equation in (19) and using (56), it is found that the scattered field for $\rho < b$ is expressed as

$$\begin{aligned}\phi(\rho,z) = {} & \frac{1}{(2\pi)^{1/2}} \int_{-\infty+ic}^{+\infty+ic} \left[\Psi_-(b,\alpha) \frac{I_0(\gamma\rho)}{I_0(\gamma b)} - \sum_{n=1}^{\infty} \frac{\alpha(f_n - g_n)}{\alpha^2 + \gamma_n^2} I_0\left(-\frac{i\xi_n\rho}{b}\right) \right] e^{-i\alpha(z+L)} d\alpha \\ & + \frac{1}{(2\pi)^{1/2}} \int_{-\infty+ic}^{+\infty+ic} \Psi_+(b,\alpha) \frac{I_0(\gamma\rho)}{I_0(\gamma b)} e^{-i\alpha(z-L)} d\alpha - \phi^i(\rho,z).\end{aligned} \tag{57}$$

Since the region inside the cavity is identified by $0 < \rho < b$ and $|z| < L$, the first and second integrals involved in (57) can be evaluated by deforming the contours into the lower and upper half-plane, respectively. It is found that singularities for the first and second integrals in this process of deformation are only simple poles at $\alpha = -i\gamma_n$ for $n = 1, 2, 3, \ldots$, and $\alpha = i\gamma_n$ for $n = 1, 2, 3, \ldots$, respectively. Thus, the evaluation of the integrals leads to the following scattered field representation:

$$\phi(\rho,z) = -\phi^i(\rho,z) + \sum_{n=1}^{\infty} T_n \sinh\gamma_n(z+L) I_0(-i\xi_n\rho/b), \tag{58}$$

where

$$T_n = \frac{2^{3/2}\pi^{1/2} e^{-2\gamma_n L}}{\xi_n\gamma_n I_1(-i\xi_n)} E_+(b,+i\gamma_n). \tag{59}$$

In (58), the first term exactly cancels the incident field defined by (5) as expected, whereas the second term represents the TM mode into the cavity.
Using (19), (56) the scattered field for $\rho > b$ is found to be

$$\phi(\rho,z) = (2\pi)^{-1/2} \int_{-\infty+ic}^{+\infty+ic} \Psi(b,\alpha) \frac{K_0(\gamma\rho)}{K_0(\gamma b)} e^{-i\alpha z} d\alpha, \tag{60}$$

where $\Psi(b,\alpha)$ can be expressed as

$$\Psi(b,\alpha) = \frac{i\omega\varepsilon}{\gamma^2} \left[e^{-i\alpha L} E_-(b,\alpha) + e^{+i\alpha L} E_+(b,\alpha) \right] \tag{61}$$

by making use of (9) and (20a,b). The field outside the cavity is actually represented as a combination of (57) and (60), but the region $0 < \rho < b$ outside the cavity is of less interest than the field from a practical point of view. Therefore, the derivation of the scattered far field for $0 < \rho < b$ will not be discussed here. Applying the differential relations (3) to (60) we express the field components thought the integrals. The integrands singularities of such integrals are the only brunch points at $\alpha = \pm k$. Hence we can simply apply the saddle point method by the use of the polar coordinate $z = R\cos\theta$, $\rho = R\sin\theta$ for $0 < \theta < \pi$ to derive the far field asymptotic expression of field components. For h_φ component this gives

$$h_{\varphi}(\rho,z) \sim \frac{\omega\,\varepsilon(\pi/2)^{3/2}}{ik\sin\theta\,K_0(ikb\sin\theta)} \times$$
$$\times\left[E_-(b,-k\cos\theta)e^{+ikL\cos\theta} + E_+(b,-k\cos\theta)e^{-ikL\cos\theta}\right]\frac{e^{ikR}}{R} \qquad (62)$$

for $\rho > b$ as $k\rho \to \infty$.

8. The case of semi-infinite cylinder with a plate internal termination

Let us consider a 3-D cavity formed by a semi-infinite circular waveguide with an interior planar termination, as shown in Fig. 3, where the cavity surface is perfectly conducting and of zero thickness, being defined in cylindrical coordinates (ρ, φ, z) as follows

$$S = \left\{(\rho,\ \varphi,\ z)|\rho{=}b,\ 0 \le \varphi \le 2\pi,\ z \in \begin{Bmatrix} (-\infty, L] & \text{for} & \rho = b+0 \\ [-L, L] & \text{for} & \rho = b-0 \end{Bmatrix}\right\} \qquad (63)$$
$$\cup\left\{(\rho,\ \varphi,\ z)|\ 0 \le \rho \le b,\ 0 \le \varphi \le 2\pi,\ z = -L\right\}.$$

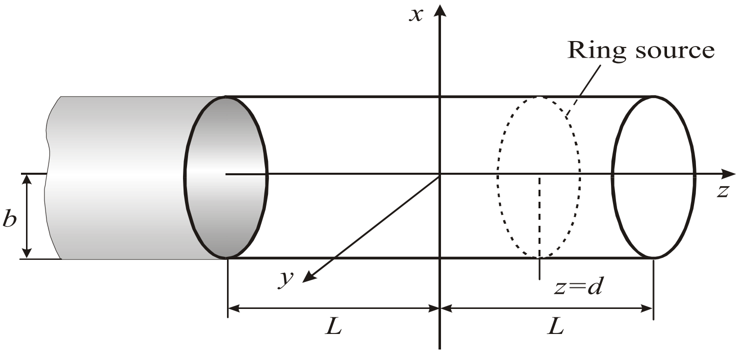

Fig.3 Semi-infinite cylinder with a plate internal termination

The structure (63) is a partial case of our previous geometry (1). The cavity (63) is assumed to be excited by uniform ring source as given in (2). Applying the upper technique we derive the transformed wave equations as follows

$$\hat{T}\Phi(\rho,\alpha) = 0 \quad \text{in} \quad \rho > b \quad \text{for} \quad |\tau| < k_2, \qquad (64a)$$

$$\hat{T}\left[\Phi_1(\rho,\alpha) + e^{i\alpha L}\Psi_+(\rho,\alpha)\right] = -\alpha\, e^{-i\alpha L} g(\rho) \quad \text{in} \quad 0 < \rho < b \quad \text{for} \quad \tau > -k_2. \qquad (64b)$$

Hence we lead this problem to the Wiener-Hopf equation with the result that

$$\widetilde{J}_1(b,\alpha) = \frac{i\omega\,\varepsilon}{b}\frac{e^{i\alpha L}E_+(b,\alpha)}{M(\alpha)} - \frac{2\omega\varepsilon\,\alpha\, e^{-i\alpha L}}{b}\sum_{n=1}^{\infty}\frac{e^{-2\gamma_n L}E_+(b,+i\gamma_n)}{\gamma_n(\alpha^2+\gamma_n^2)} + \widetilde{U}(b,\alpha). \qquad (65)$$

Here $\widetilde{J}_1(b,\alpha)$ denotes the Fourier integral of the unknown current on the semi-infinite cylindrical surface over the interval $-\infty < z < L$, and

$$\widetilde{U}(b,\alpha) = H^i(b,\alpha) - J_+(b,\alpha). \qquad (66)$$

Next, multiplying both sides of (65) by $e^{-i\alpha L}M_-(\alpha)$ and following a procedure similar to that employed to obtain (39) yields,

$$E_+(b,\alpha) - M_+(\alpha)\sum_{n=1}^{\infty}\frac{e^{-4\gamma_n L}M_+(i\gamma_n)E_+(b,i\gamma_n)}{i\gamma_n(\alpha+i\gamma_n)} = M_+(\alpha)R_+(\alpha)\,. \qquad (67)$$

Now (67) gives the exact solution of the Wiener-Hopf equation (65), and is more simple because it does not contain the brunch cut integral with unknown integrand. Scattering field representation can be expressed here as in Section 7. For the far field evaluation we can receive the formula similar to (62) which keeps only the term $E_+(-k\cos\theta)e^{-ikL\cos\theta}$ in the square brakes.

9. Conclusion and remarks

In this paper, we have analyzed the axial symmetric diffraction problem for a circular waveguide cavity rigorously using the Wiener-Hopf technique. The method of solution is a generalization of the approach we have established previously for the analysis of the diffraction by a parallel-plate waveguide cavity [13, 19], and it involves the use of the finite Fourier-Bessel series in the formulation. It is to be noted that (37) and (39) are the key results which give the exact solution of the Wiener - Hopf equation (30) but in formal sense because they involve infinite series with unknown coefficients and infinite brunch-cut integrals with the unknown integrands. For investigating the axial symmetric electromagnetic fields scattered by the cylindrical waveguide cavity numerically approximate procedures and approximate solution of the Wiener - Hopf equation are proposed. Taking the Fourier inverse of the solution in the transform domain we have derived the scattered field inside and outside the cavity. In particular, the h_φ far field component outside the cavity has been evaluated using the saddle point method of integration. The case such as the wave diffraction problem for the semi infinite cylinder with internal plate termination has been considered and the key equations have been derived. The application of the upper developed technique to the more complicated vector diffraction problems is given in [21].

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