Modeling of light scattering from waveguide irregularities by Beam Propagation Methods

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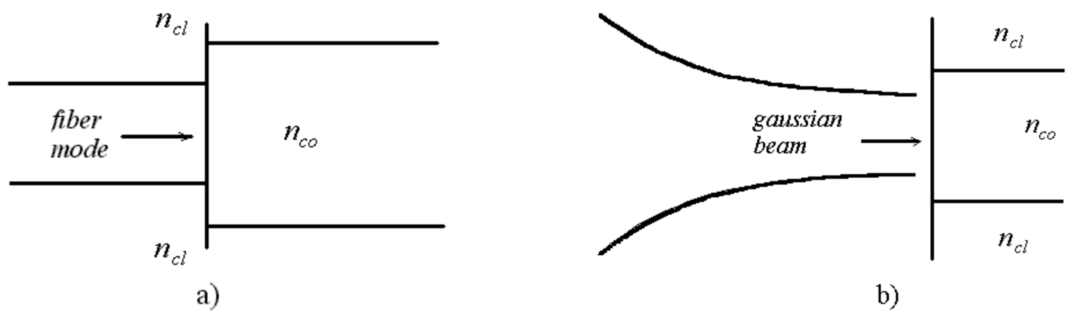
Abstract *Spatial transient process of the radiation field propagation in irregular optical waveguides is simulated numerically by Beam Propagation Methods (BPM). The results obtained by the Fast Fourie Transform - BPM, the Finite Differences - BPM and the Collocation Method applied to treat planar waveguide are compared and analyzed. The FD-BPM is shown to be the most efficient numerical technique to model the light scattering from irregularities if a waveguide can be considered as the weakly - guiding one. Spatial transient processes of* LP_{02} *mode propagation through the cutoff cross-section of sharp and smooth discontinuities of optical fibre are simulated by FD-BPM.*

Introduction

If a radiation propagating along a waveguide axis occurs to be mismatched with modal field in some cross-section, the so-called spatial transient process appears which actually is a result of the light scattering from the irregularity. Inside the transient region the total field can be expressed as a sum of discrete guided modes and continuum of radiation modes. Amplitude of the modes and character of their interaction depends on the type of the irregularity. Two principal types of irregularities are shown in Fig1, Fig.2.

We consider below the step-like discontinuity made of two co-axial waveguides with different core radii joined consecutively (Fig.1,a), as well as unmatched excitation of a waveguide mode by a gaussian beam (Fig.1,b). When a guided mode reaches the step-like discontinuity, it is mismatched in the plane of the step irregularity. As a result, forward - and backward - scattering fields appear, which occure to be caught partially by the waveguides and propagate as guided modes in both directions.



.Fig.1. Step-like discontinuity.

Different computational techniques have been used to calculate transmittance and reflectance of the step-like discontinuity of the type (a) including the mode matching approximate method [1] and the method of integral equations [2]. In the weakly-guiding approximation (n_{co} - n_{cl} << n_{co}), the backscattered power can be neglected and the problem can be simplified to one-direction scalar beam propagation.

Numerical simulations of the spatial transient process in planar waveguide, which appears as a result of unmatched excitation of the step-index structure by gaussian beam (Fig.1,b), have been done in [3] using Beam Propagation Method (BPM). The length of the transient process was evaluated and spatial distribution of the radiation field was plotted.

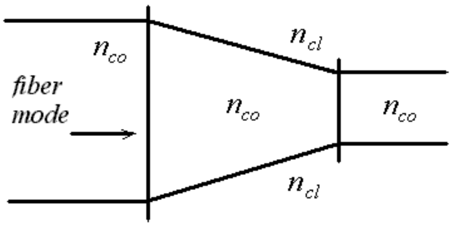


Fig.2. Taper

In smooth irregular structures (tapers, Fig.2), the spatial transient regime is continuous, and depends on the taper adiabaticity [4]. The local mode approach is based on the approximation of a taper by the consequence of longitudinally uniform segments [5]. The refraction index profile is assumed to be the one of a regular waveguide defined at the given longitudinal coordinate for every segment. The phase of a local mode is approximated by averaging the longitudinal propagation constant. The local modes have been proved to be suitable only in adiabatic approximation of the field propagation in the waveguides with slowly varying parameters. This approach was generalised on the non-adiabatic case by the coupled-mode theory initially used by Marcuse [6]. It has been used to analyse nonuniform slab waveguides [7] and optical fibres [8].

To analyse waveguide tapers and step-like structures one can use the Step-Transition Method [9] which was initially developed by Marcuse to analyse low-loss tapers. This technique was modified so that it could be used to analyse any taper and step-like structures. It is based on the matching procedure of the transverse fields across the step. A similar Mode-Matching procedure have been developed that in addition to discrete set of guided modes in sub parts of the cross-section includes also continuous spectra of radiation modes [10]. The method starts by dividing the total cross-section into laterally uniform sections. For each section the complete set of modes is set up. By matching tangential field components at the interface to those of the complete set of modes in the neighbouring section a scattering matrix is constructed which relates the mode amplitudes which are excited in the neighbouring section and the amplitudes of the reflected modes in the section of incidence to the incident mode amplitudes. For the numerical evaluation of the integrals over continuous spectra of radiation modes in this formulation suitable discretization and normalisation approximates the radiating modes by discrete set of modes.

The most commonly used numerical methods to simulate light propagation in optical waveguides are modifications of beam propagation method (BPM) originally developed by Fleck, Morris and Feit [11]. The so called FFT-BPM based on the Fast Fourier Transform (FFT), proved to be an accurate and efficient tool for solving a variety of propagation problems in waveguide geometry involving one or two transverse Cartesian coordinates [12-14]. An alternate numerical scheme to solve the wave equation is to use a finite difference (FD) approximation [15]. Following the Finite - Difference Beam Propagation Method (FD-BPM) the wave equation is replaced by a finite - difference scheme. The resulting three - diagonal system of equations is solved by some iterative procedure.

Other method for studying the propagation of a field through the general waveguiding structure was developed in [16]. It is based on converting the scalar wave equation into a matrix total differential equation using the Collocation Method (CM), which may be stated as follows. The partial differential equation is assumed to be satisfied exactly at some points along the radial coordinate. These points are known as the collocation points. The total field is expressed as a linear combination of a set of orthogonal functions. So, by applying the collocation principle, the scalar wave equation can be converted into a set of total differential equations, which can be solved then numerically using any standard procedure, such as the Runge-Kutta method.

The main goal of this paper is to analyze some numerical techniques useful to simulate the spatial transient process of beam propagation through the guiding structures described above. As an example, planar waveguide is considered being excited by a gaussian beam. Three modifications of the Beam Propagation Method are applied: the FFT-BPM, the FD-BPM and the CM. Transformation of LP_{02} guided mode into a radiation field in cylindrical optical fiber below the cutoff cross-section is simulated by the FD-BPM using sharp and smooth discontinuities.

Radiation field expansion

The amplitude of the total time-harmonic scalar field propagating along the z-axis of an irregular linear waveguide can be written as a sum of the discrete set of N_g guided waves and the radiation field [17,18] (in the case of small discontinuities in the weakly-guiding structure, the backscattering can be neglected):

$$E(x,z) = \sum_{n=1}^{N_g} A_n(z)E_n(x)\exp(i\beta_n z) + E_{rad}(x,z), \tag{1}$$

$$E_{rad}(x,z) = \sum_{j=1}^{\infty} \int_0^{\infty} dw A_j(w,z)E_j(w,x)\exp(i\beta_j(w)z) \tag{2}$$

with A_n and β_n being the amplitudes and the longitudinal wavenumbers of the *n-th* guided mode, A_j and β_j being the amplitudes and the longitudinal wavenumbers of the *j-th* radiation mode. The functions $E_n(x)$ and $E_j(w,x)$ describe transverse distributions of the guided and the radiation modal fields, respectively. In the case of cylindrical waveguide the integrands $E_j(w,x)$ are meromorphic functions on the multi-sheet Riemann-surface of the complex parameter w, which is treated as the cladding transverse wavenumber of a fiber mode [18]. The poles of the functions $E_j(w,x)$ correspond to the leaky modes and are located in the w - plane depending on the value of the characteristic frequency $V = ka\sqrt{n_{co}^2 - n_{cl}^2}$ determined in the given cross-section of the fiber (here $k = 2\pi/\lambda$ is the free-space wavenumber, a is the radius of the fiber core, n_{co} and n_{cl} are the refraction indices in the core and in the cladding, respectively). Each pole formally is a solution of the characteristic equation of the eigenvalue problem and characterizes the field which increases exponentially in the radial direction.

Extraction of the leaky modes from the integral (2) is reasonable in some limited spatial domain near the waveguide axis. The field here can be described mainly by the residues, the remaining integral being very small. Outside the domain, the field increases in the radial directions. However, the remaining integral increases as well, but has the opposite sign. The resulting field is finite. So, far from the waveguide axis, the leaky modes extraction becomes unreasonable.

Modeling of the radiation field in planar waveguide

Two different Beam Propagation Methods have been applied to simulate the radiation field propagation in a step-index planar waveguide with infinite cladding excited by the Gaussian beam. One is the FFT-BPM used to solve the Helmholtz equation:

$$\left(\frac{\partial^2}{\partial z^2}+2i\beta\frac{\partial}{\partial z}+\frac{\partial^2}{\partial x^2}+\chi\right)F(x,z)=0 \qquad (3)$$

for the slowly varying amplitude of the total field $F(x,z)$ [3].The other is the FD-BPM [19] applied to solve the paraxial parabolic wave equation:

$$\left(2i\beta\frac{\partial}{\partial z}+\frac{\partial^2}{\partial x^2}+\chi\right)F(x,z)=0\,, \qquad (4)$$

in assumption that the paraxial approximation is valid for a weakly-guiding waveguide. The FD-BPM algorithm is known to fail being applied to solve the Helmholtz equation. Otherwise, the FFT-BPM is free of this disadvantage. On other hand, the FFT-BPM is sensitive to refraction index difference in transverse direction. Therefore the FFT-BPM breaks down when applied to structures with large index discontinuities.

Propagation of the radiation field excited by a gaussian beam on an input endface of a single-mode planar waveguide was simulated numerically. The beam was assumed to be parallel to the waveguide layers. The excitation have been considered to be symmetric (beam axis coincides with the input endface center) as well as asymmetric (beam axis is shifted out of the input endface center). The transverse distribution of the incident radiation field was taken as a difference between the incident field distribution and the modal one [3]:

$$E_{rad}=E_{inc}-E_{\mathrm{mod}}, \qquad (5)$$

where E_{mod} is proportional to a fundamental guided mode distribution Ψ_0 normalised to the unit power flow, with a complex amplitude α:

$$E_{\mathrm{mod}}=\alpha\Psi_0. \qquad (6)$$

The refractive indices chosen for the guiding and the cladding layers were $n_1 = 1.491$, $n_2 = 1.479$, respectively. The total width of the guiding layer was $D = 2.3\mu m$. The wavelength was $\lambda = 1.53\ \mu m$.

The FFT-BPM computational parameters were chosen in such a way that a part of the nonpropagating (evanescent) spectrum and the whole propagating spectrum of the field were included in the k_y-space computational window in order to ensure exact modeling of the field propagation [3]. The mesh size was $\Delta x=0.25\ \mu m$ and the longitudinal step size was $\Delta z=0.5\ \mu m$.

The FD-BPM method has been based on the Crank-Nicholson scheme and an iterative procedure proposed in [20]. The mesh size was $\Delta x=0.25\ \mu m$ and the longitudinal step size was $\Delta z=2.5\ \mu m$.

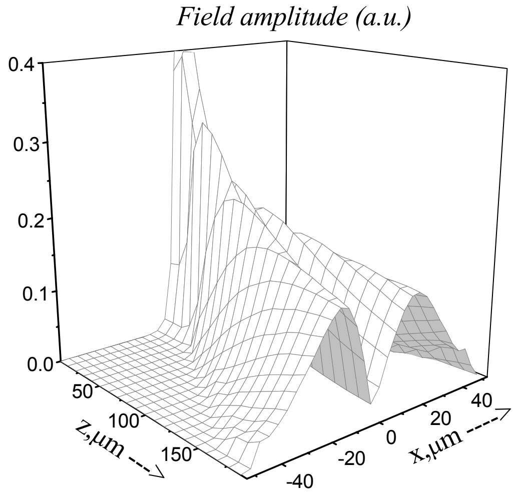
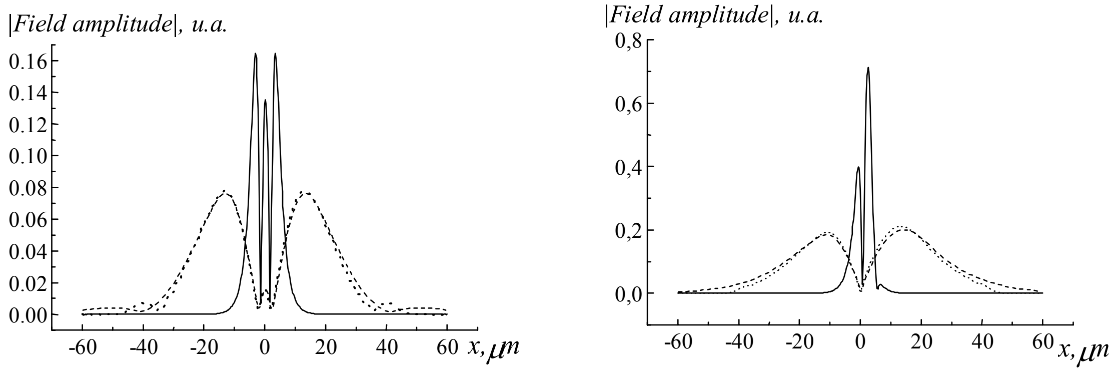


Fig.4

Spatial transient regime of the radiation field propagation is shown in Fig.4 simulated by the FD-BPM. The radiation field profiles at the input endface of the waveguide and the ones calculated after a propagation distance of 150 μm are presented in Fig.5 a,b. The results make evidence that the spreading of the paraxial beam (dotted line, FD-BPM) is less than the wide-angle one (dashed line, FFT-BPM). The discrepancy depends on the excitation conditions and grows with the shift of the input beam relative to the input endface center.



a) b)

Fig.5

Additionally, the calculation time required to simulate the radiation field propagation by both these methods was evaluated (Fig.6). The computation efficiency of the FD-BPM simulations is proportional to the number of grid points N while the one of the FFT-BPM is of the order NlogN [21]. For comparable accuracy much smaller longitudinal step is to be used in the FFT-BPM than in FD-BPM. This is one reason for the greater calculation time of FFT-BPM to achieve the same accuracy. The other reason is that one has to use many FFT expansion terms to describe a complicated spectrum of the radiation field.

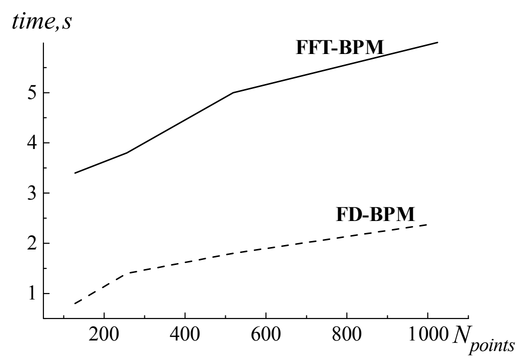


Fig.6

This planar guiding structure was treated by the CM with the expansion functions taken as the proper functions of the parabolic index waveguide (Hermite-Gauss functions). We have calculated the condition numbers of the propagation matrices depending on the parameters of the expansion functions and on the number of collocation points. The values of the condition numbers of the matrices are about 500

The condition number versus the width *w* of the gaussian term of the expansion functions are shown in Fig.7. When the greater number of collocation points was used the condition number decreased. However, for large number of collocation points, it was necessary to make hard matrix inversions. Moreover, the longitudinal step should be about 0.01 of the diffraction length. The method is suitable to treat a guiding structure, if the proper functions of this structure are used as the expansion functions. However it requires a complicated modification of the program for each index profile.

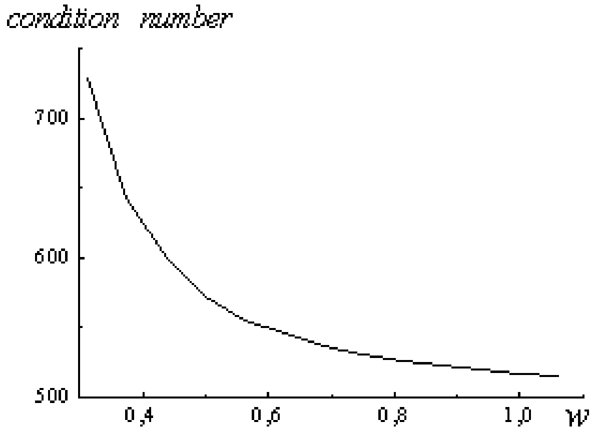


Fig.7

Conclusions: Beam Propagation Methods based on the expansion of total field in a set of plane waves (FFT-BPM) or modal functions (CM) are shown to be nonefficient to describe the complicated structure of wide-spreading radiation field. The FD-BPM seems to be the most efficient and universal technique to simulate the radiation field propagation provided that the weakly-guiding approximation can be used.

Modeling of a fibre mode propagation through the cutoff cross - section

The FD-BPM was used to simulate the transformation of the LP_{02} mode when it was propagating through the cutoff cross-section. The model of a down-tapered fiber with the step-index profile of refraction index and infinite cladding have been used first to simulate quasi-adiabatic ppropagation. A taper with the length L and linear longitudinal profile: $a(z) = a_1 - \Delta a\ z / L$ was considered. Transverse profile of the field at the input end-face of the taper corresponded to the guided LP_{02} mode of a regular step-index fiber with the fiber core radius a_1. The fiber mode was supposed to be excited by the light source with the wavelength λ = 1μm. The taper parameters were chosen to be as follows: L = 1mm, a_1 = 5μm, Δa = 1μm. The corresponding range of the characteristic frequency variation was $3.7 < V(z) < 4$ containing the cutoff value (for the lossless fiber $V_c \approx 3.83$). When the fiber radius decreases the guided mode reaches the cutoff cross-section of the taper, passes through it and forms a spatial wave which leaks outside the fiber core (Fig.8). However, a part of radiation, which propagates below the cutoff closely to the fiber core, is of the same transverse distribution as the guided mode was.

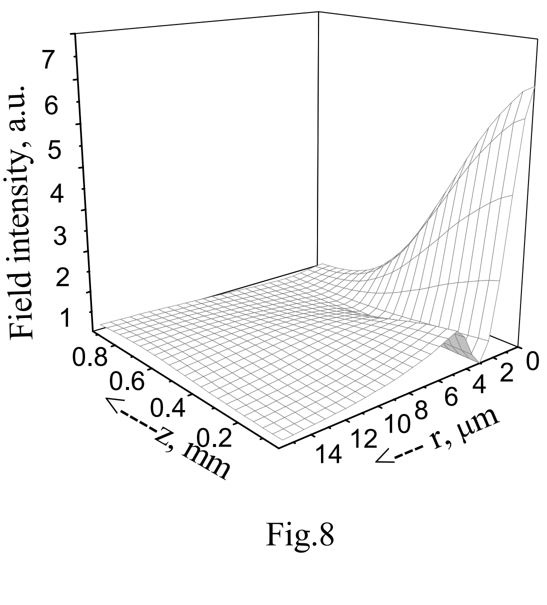


Fig.8

The total field can be sufficiently described here by the leaky mode field. It propagates over some long distance along the fibre core since just below the cutoff the imaginary part of the longitudinal propagation constant is small: $\beta'' \approx 10^{-5}\ \mu m^{-1}$. Far from the fiber axis the radiation field is finite and goes to zero at infinity.

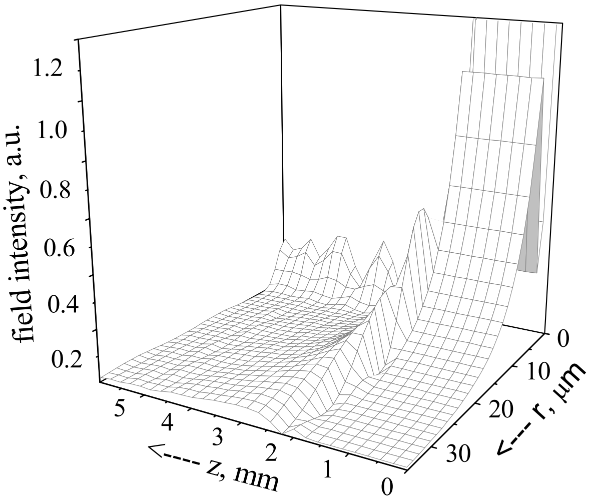


Fig.9

The non-adiabatic propagation of the total field through the step-like discontinuity is shown in Fig.9. The radius variation, the excitation conditions and other parameters of the structure were the same as of the taper. Transmittance of the guided part of the field calculated by matching the modal fields in the plane of discontinuity is about 4% [22]. The major part of the LP_{02} mode then transforms into the radiation field. The beatings between the fundamental mode and the radiating part of the field near the fiber core are clearly seen. The long length of the beatings is predominantly due to the spatial wave, which attenuates along the fiber axis as $z^{3/2}$ [17]. The spatial wave propagates outside the guiding region in all directions and forms a complicated interference field around the fiber core for propagation distances up to 10 mm.

Conclusions

Numerical simulations of the radiation field propagation are a hard problem because of the complicated form and the wide-spreading character of the field. The computational window should be large enough for the boundary conditions would be fulfilled. If the numerical scheme is sensitive to the value of longitudinal step, or needs a great number of expansion functions, it is not efficient. Alternatively, the FD-BPM works with the total electric field and automatically includes the effects of both guided and radiation field as well as mode coupling and conversion. It provides an efficient tool to simulate the spatial transient process of the radiation field propagation in weakly-guiding irregular structures.

Behavior of the total field inside the transient region depends on the type of irregularity. The shape of the outgoing wave, as well as the leaky mode contribution into the total field, depends on the value and on the longitudinal variation of the fiber characteristic frequency. Radiation field propagation in quasi-adiabatic smoothly varying structures can be efficiently described near the fiber axis by the leaky mode field. Otherwise, in sharply varying structures, the spatial wave propagation is dominant. The length of the spatial transient process is determined here by the interference of the spatial wave and guided part of the total field. The length can be up to several tens of millimeters.

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