A Hybrid UTD-MoM Analysis of the Radiation from Large, Finite Planar Arrays with a Tapered Excitation

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Abstract: *A hybrid method which combines the asymptotic high frequency uniform geometrical theory of diffraction (UTD) ray concept with the numerical moment method (MoM) has been developed to provide an efficient analysis of the electromagnetic radiation from electrically large, finite, planar periodic arrays. In this hybrid UTD-MoM approach, the number of unknowns to be solved is drastically reduced as compared to that which is required in the conventional MoM approach. This substantial reduction in the unknowns is made possible by introducing only a few appropriate UTD type global basis functions to describe the functional form of the unknown array currents everywhere except very near the array boundary. The utility of this hybrid approach is demonstrated here for the simple case of a large, planar rectangular phased array of thin metallic halfwave dipoles in air, with a tapered excitation. Some numerical results are presented to illustrate the efficiency and accuracy of this hybrid method.*

Introduction

A hybrid method, which combines the asymptotic high frequency based UTD ray concept with the numerical MoM approach has been developed to provide a relatively efficient analysis of EM radiation/scattering from an electrically large, planar, periodic, finite array [1]. An extension to the hybrid UTD-MoM approach of [1] is presented to deal with not only uniform but also tapered array excitations [2]. In this hybrid UTD-MoM method, an integral equation formulation for the unknown currents on the array elements is solved within the MoM framework in a relatively efficient manner by essentially introducing only a few appropriate UTD-type global basis functions to describe the functional form of the unknown array currents that are valid everywhere except very near the array boundary. Consequently the number of unknowns to be solved via this hybrid UTD-MoM approach is drastically reduced as compared to conventional MoM approach which requires the number of unknowns to be typically equal to or greater than the number of elements in the large array. Additional relations can be invoked which relate the behavior of the array currents even near the array boundary, with the exception of array corners, thereby further reducing the number of unknowns. It is evident that numerical methods, such as MoM, when used alone in a conventional fashion, become rapidly inefficient for solving large finite array problems due to the extremely large number of unknowns that need to be solved in such methods. Although asymptotic high-frequency methods such as the UTD are primarily useful for analyzing electrically large problems, the large-array geometry in general will contain electrically small antenna elements which cannot be handled by high frequency methods alone. It is therefore of interest in this work to systematically combine best features of asymptotic high-frequency and numerical methods in a hybrid fashion to overcome the limitations of each, for the efficient analysis of large finite array radiation problems.

Most previous works on analysis of large arrays employ the theory of periodic structures to study the radiation/scattering from planar periodic arrays of infinite extent with periodic excitation. Of course, the theory of periodic structures cannot be directly applied in its original form to solve the finite array problem. In one of the earliest attempts, the finite array solution was expressed as a convolution of the infinite array solution with the tapered finite array excitation by Ishimaru *et al.* in [3], and was later extended to deal with planar rectangular arrays of microstrip patches in [4]. While the approach in [3,4] provides a more efficient numerical solution than conventional MoM, it still requires the solution of a very large number of MoM unknowns. More recently, a highly efficient ray solution (GTD) based on the geometrical theory of diffraction was obtained for the fields scattered by linear array of finite width upon employing a truncation of the infinite array excitation by Felsen *et al.* [5]. UTD solutions have also been obtained by the present authors for the development of a hybrid UTD-MoM analysis of finite planar arrays in [1]. More extensive uniform asymptotic HF ray solutions including diffraction of evanescent Floquet modes have been developed in [6,7]. It is noted that all of the GTD/UTD asymptotic ray solutions are based on the Kirchhoff approximation which assumes that the aperture field distribution over the finite array is same as that for the corresponding infinite array. The truncation of the Kirchhoff integral at the boundary of the finite array then provides a ray description for the array edge and corner diffraction effects when this integral is evaluated asymptotically for high frequencies, provided that the observation point is not too close to the array edges and corners. Although the Kirchhoff approximation cannot provide accurate results for the input impedance of antenna elements which are close to the array edges, it nevertheless provides valuable information on the functional behavior of the array current distribution away from the immediate vicinity of the array edges and corners. This information on the functional behavior of the array current distribution is crucial for drastically reducing the large number of unknowns in the conventional MoM approach, thereby providing the basis for a far more efficient hybrid UTD-MoM solution to deal with the electrically large array problem. A similar, yet sufficiently different hybrid solution is given in [8]; an important difference between [8] and [1,2] (as well as this paper) is the choice of UTD basis set as will be indicated later.

Typically the array current distribution is quite different from the tapered feed excitation especially near the array edges and corners. Hence, once the array current distribution is obtained by the hybrid method, a DFT spectrum of the nonuniform array current distribution is next used to evaluate the array near/far fields using the UTD [9] which is valid and accurate for each DFT component. Furthermore, only a relatively small number of DFT components are needed to find the near/far fields of the array via the UTD.

Formulation

A rectangular planar periodic array of identical, halfwave and thin, perfectly conducting wire dipole elements oriented in $\hat{y}$ direction at $z = 0$ in air, as shown in Figure 1, is analyzed here for the sake of simplicity in demonstrating the hybrid concept. However, this approach can be extended to deal with more complex array elements, e.g. slots, strip dipoles, patches, etc., which may be placed in planar layered structures.

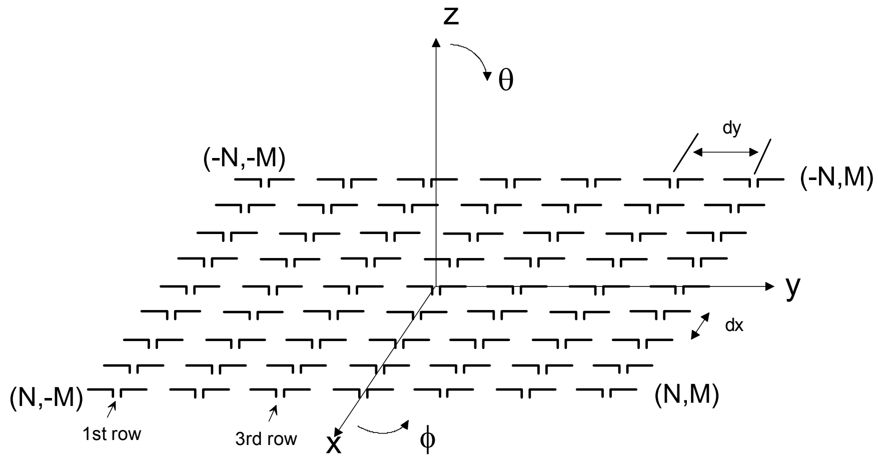

Figure 1. A planar, periodic rectangular array of (2N+1)(2M+1) identical halfwave and thin y-directed center-fed dipole antennas in air

The electric field generated by the currents on dipole array can be written, for an $e^{j\omega t}$ time convention which is assumed and suppressed, as follows

$$\overline{E}(\overline{r}) = -j\omega\mu \sum_{n=-N}^{N} \sum_{m=-M}^{M} \int_{L_{nm}} \overline{\overline{G}}_0(\overline{r} \,|\, \overline{r}'_{nm}) \cdot I_{nm}(y')\, \hat{y}\, dy' ,$$

where $\overline{\overline{G}}_0(\overline{r} \,|\, \overline{r}'_{nm})$ is the usual free space electric dyadic Green's function, $\overline{r}$ and $\overline{r}'_{nm}$ denote the position vectors for the field point and of any point on *nm*th dipole element, respectively, and $I_{nm}(y')$ denotes the current distribution on *nm*th dipole.

The integral equation for the unknown currents I_{nm} is then obtained as usual by requiring the total electric field to be zero at $\overline{r}_{pq}$ within each *pq*th perfectly conducting dipole element

$$j\omega\mu \sum_{n=-N}^{N} \sum_{m=-M}^{M} \int_{L_{nm}} \overline{\overline{G}}_0(\overline{r}_{pq} \,|\, \overline{r}'_{nm}) \cdot I_{nm}(y')\, \hat{y}\, dy' = V_{pq}\, e^{-j\beta_x p d_x}\, e^{-j\beta_y q d_y}\, \delta(y - q d_y)$$

where

$$V_{pq}\, e^{-j\beta_x p d_x}\, e^{-j\beta_y q d_y}\, \delta(y - q d_y)$$

is the impressed voltage at the center of each *pq*th dipole.

Let $\quad I_{nm}(y') \cong A_{nm}\, f_{nm}(y')$,

where $f_{nm}(y')$ is the local basis function for the half wavelength dipole and it is given by

$$f_{nm}(y') = P_{nm}(y') \sin\left[k\left(\frac{l}{2} - \left| y' - m d_y \right| \right) \right],$$

$$P_{nm}(y') = \begin{cases} 1, & L_{nm} \equiv l = \text{dipole length} \\ 0, & \text{otherwise} \end{cases}$$

and A_{nm} is the unknown coefficient (corresponding to the current at the *mn*th feed point) to be determined via the MoM. Here, as in Galerkin's method, one may select the test function to be the same as the local basis function; hence

$$\sum_{n=-N}^{N}\sum_{m=-M}^{M} A_{nm}\left[j\omega\mu \int_{L_{pq}} dy \int_{L_{nm}} dy' f_{pq}(y)\hat{y}\cdot\overline{\overline{G_0}}\left(\bar{r}_{pq} \mid \bar{r}'_{nm}\right)\cdot\hat{y}\, f_{nm}(y')\right] = V_{pq}\, e^{-j\beta_x p d_x} e^{-j\beta_y q d_y} .$$

Then, the usual MoM matrix equation can be obtained by allowing p and q to take on values $-N \le p \le N$ and $-M \le q \le M$. When N and M are large, the number of unknowns becomes very large and the order of impedance matrix also becomes very large; hence the numerical solution of this matrix equation becomes highly inefficient for large arrays.

A drastic reduction in the number of unknowns A_{nm} is possible if one employs UTD ray concepts [1], this serves to make the MoM approach faster and significantly more efficient. An asymptotic high frequency analysis of the uniform array radiation/scattering problem can show that the field at an observation point sufficiently far from the array boundaries can be viewed as a superposition of the fields of just a few rays arising from a specific set of interior and boundary points of the array. Most importantly, the number of significant rays is independent of the physical size of the large array for a given frequency. The ray fields arising from the interior of the finite array are associated with the usual periodic structure Floquet modes that would exist on the corresponding infinite periodic array. The part of the scattered field described by just the Floquet modes follows the usual geometrical optics incident ray path only for the dominant Floquet mode, whereas they follow a different type of ray path, in an extended geometrical optics sense, to the observer for any higher order propagating Floquet modes. Additional, nonpropagating Floquet modes exist which are evanescent normal to the array face but propagate along the array surface. It is noted that Floquet modal wave contributions vanish within certain regions of space surrounding the finite array if they arrive there along rays whose points of origination lie on the artificial extension of the actual finite array. Thus the finiteness of the array introduces ray shadow boundaries for the Floquet modal field contributions. The Floquet modes undergo diffraction at the array boundaries (at the array edges and corners). Furthermore, a uniform asymptotic analysis for the finite periodic arrays within the Kirchhoff approximation provides a UTD ray picture for the radiation/scattering from arrays in which the Floquet mode based edge diffraction contribution compensates for the discontinuity in the geometrical optics type incident/reflected Floquet ray fields across their shadow boundaries. The Floquet mode based corner diffraction contribution compensates for the discontinuity in the Floquet edge diffracted fields that occur because there is no edge to diffract from past a corner which truncates that edge. Thus, the asymptotic high frequency behavior far from and also very close to the finite array can be expressed as a superposition of the fields of just a few rays.

With the UTD picture in mind, the behavior of A_{nm} within the array interior can be thought of as consisting of the values directly proportional to the impressed (or excitation) array values, together with the modification resulting from the effect of edge and corner diffraction due to finiteness of the array. The edge and corner diffraction effects in A_{nm} are assumed to have the same functional form as the fields, namely a conical wave diffraction from the edge and a spherical wave diffraction from the corner, except within their shadow boundary transition regions.

As prescribed in [1], the array can be divided into a large inner part, and a thin, outer boundary part. Then, one can describe the functional variation of A_{nm} using global UTD based approximation within an array interior as follows

$$A_{nm} = \begin{cases} C_{-n+N+1,\,m+M+1} & \text{for corner cells of the outer part} \\ \mathrm{E}_{-n+N+1,\,m+M+1} & \text{for edge cells of the outer part} \end{cases}$$

and

$$A_{nm} \sim \left\{ D \frac{V_{nm}}{V_{00}} e^{-j\beta_x n d_x} e^{-j\beta_y m d_y} + \sum_{e=1}^{4} \left(\left[\frac{A_e}{\sqrt{s_e}} + \frac{B_e}{\sqrt{s_e^3}} + \frac{F_e}{\sqrt{s_e^5}} \right] V_e(x_e, y_e) e^{-jks_e} e^{-j\beta_x x_e} e^{-j\beta_y y_e} \right) \right\}$$

for the inner part,

where β_x, β_y is the impressed (or excitation) phase.

It is noted that the UTD based part of A_{nm}, which depends directly on the impressed array excitation, is described by the coefficient of proportionality D, whereas the UTD part of A_{nm} which depends on edge diffraction effectively is described by the far fewer unknown coefficients A_e, B_e and F_e. Also, s_e denotes the ray distance along the edge diffracted field from the eth edge and $\hat{s}_e$ depends only on the values of β_x and β_y. The $\frac{V_{nm}}{V_{00}}$ denotes the effect of the array feed taper on the element current distribution over the array. V_e denotes the taper along the direction parallel to the edge. It is noted that the UTD basis here does not involve a sum on Floquet modes, whereas that in [8] does. The results for only uniform feed tapers have been shown in [1] and the hybrid solution was shown there to compare very well with the conventional MoM solution. For instance, for the 50×50 element array, the conventional MoM would require 2500 unknowns, whereas hybrid UTD-MoM would require only 81 unknowns for uniform excitation. Also, the number of unknowns remains essentially unchanged in the hybrid method even if the physical size of array is increased. In [1], the $F_e = 0$; however, for tapered excitation, F_e must be included for increased accuracy which results because it essentially provides an additional degree of freedom for the MoM solution.

A further reduction in unknowns is possible since it follows again from a local UTD viewpoint that

$$\mathrm{E}_{-n+N+1,\, m+M+1} \cong E_Q^{\mathrm{mid}} \frac{\mathrm{V_e}(-n+N+1, m+M+1)}{\mathrm{V}_{00}} e^{-j\beta_x(-n+N+1)d_x} e^{-j\beta_x(-m+M+1)d_y}.$$

The index Q implies that either n or m are fixed for a given linear array formed by the edge cells of the outer part which run parallel to the physical edge.

Numerical Results

Numerical results obtained by the hybrid UTD-MoM approach for tapered array feed excitations are presented in this section. Typically the array current distribution is quite different from distribution corresponding to the voltage excitation, especially at the feed points for elements at and near array edges and corners. Hence, once the array current distribution is obtained by the hybrid method, a DFT of the array current distribution is used to evaluate the array near/far fields using UTD, [9]. There are two important advantages in using DFT representation of array current amplitudes. The first one is that the DFT spectrum of practical array currents are very compact, hence only a few number of DFT terms are usually sufficient. Secondly, near/far fields can be found in closed form via UTD for finite arrays, which although not directly applicable to arbitrary array current distributions is valid for each DFT component of the current and hence this DFT based UTD provides a useful physical insight into the array radiation mechanism as opposed to the conventional element by element field superposition approach.

The radiation patterns of 41×41 y-directed halfwave dipole array are shown in Figure 2. In this case, the feed distribution is assumed to be a Taylor distribution in x and uniform in y. Figure 3 illustrates the normalized element currents for the 1st and 26th rows. The number of unknowns is only 61 in the present hybrid method, whereas 1681 unknowns are required in the conventional MoM approach. Also, the number of unknowns remains unchanged in the hybrid method even if the physical size of array is increased. There is a good agreement between the hybrid solution and reference solution based on the conventional MoM.

Work is currently in progress to include different antenna types and the effects of material substrates/superstrates.

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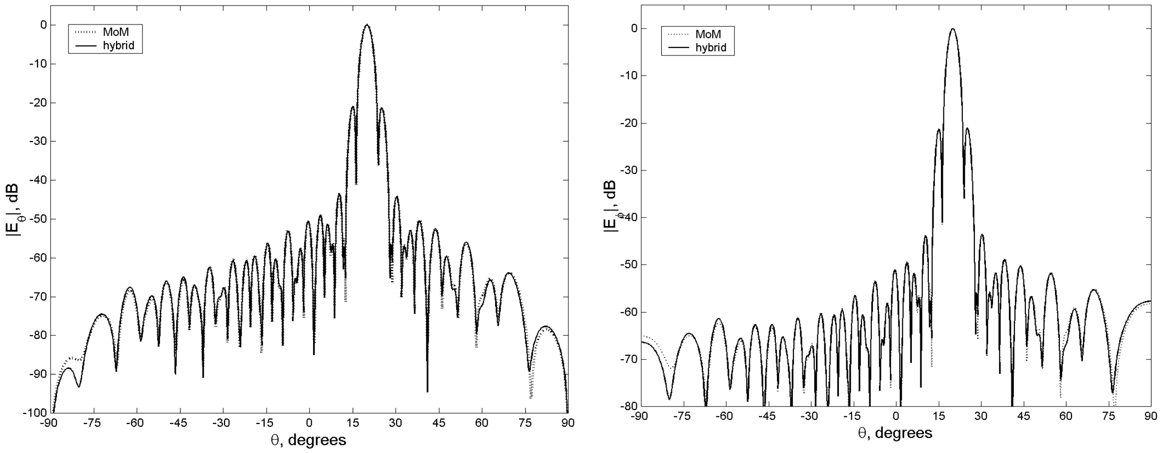
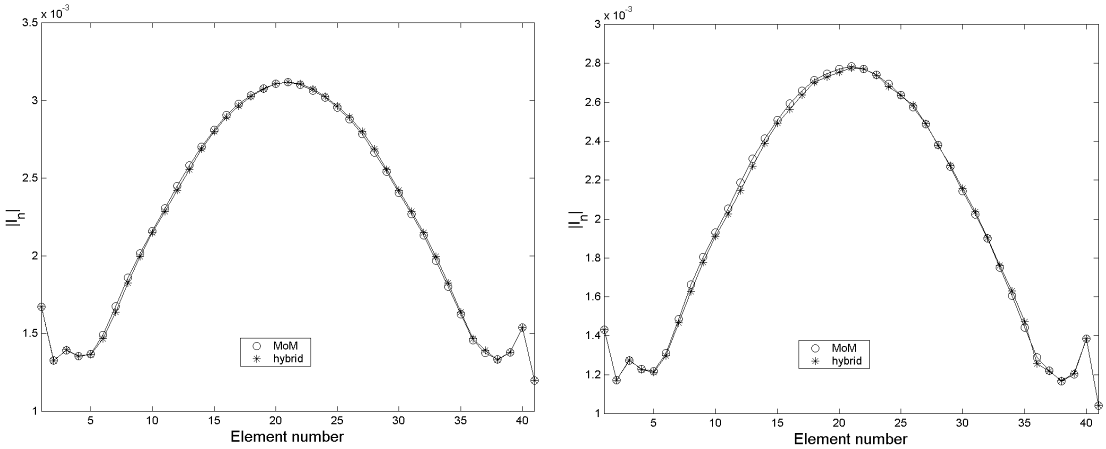


Figure 2. Radiation patterns for 41×41 dipole array, d_x=0.3λ, d_y=0.6λ, scan direction (θ=20°,ϕ=40°), feed: Taylor distribution in x (SLL=25dB), uniform in y.



1st row 26th row

Figure 3. Current amplitudes for 41×41 dipole array, d_x=0.3λ, d_y=0.6λ, scan direction (θ=20°,ϕ=40°), feed: Taylor distribution in x (SLL=25dB), uniform in y.