DOMAIN PRODUCT TECHNIQUE ANALYSIS OF ELECTROMAGNETIC SCATTERING FROM MULTIANGULAR CYLINDRICAL STRUCTURES

Vitaliy P. Chumachenko

The author is with Electronics Engineering Department, Gebze Institute of Technology, 41400 Gebze, Kocaeli, Turkey, on leave from Technical University of Zaporizhzhya, 64 Zhukovsky Str., Zaporizhzhya, 330063 Ukraine

Abstract - *Domain product technique solution is reported for the problem of scattering from metallic and penetrable cylindrical structures with arbitrarily polyhedral boundaries. Transverse magnetic and transverse electric excitations are treated. In both the interior and exterior regions, the sought-for quantities are expressed in the forms of Mathieu function series. Boundary value problem is reduced to a system of infinite matrix equations with respect to expansion coefficients. Solution to the system is found using truncation procedure. The technique gives an opportunity for a substantial variation of the geometry of the structure without applying other mathematical models. It is efficient in middle frequency range and enables accurate numerical analysis of fairly complicated objects.*

1. Introduction

The problem of scattering by cylinders has many specific applications. The literature on this subject is very extensive. Of many books and papers, we mention only [1-11] discussing the problem and giving references. Currently, finite-element method (FEM), finite-difference method (FDM) and hybridizations of those with low-frequency and high-frequency techniques are the most promising methods for large scale simulation without placing restrictions on the geometry and material composition of the structure [12]. However, for certain configurations, the method of moments and analytical techniques remain the most accurate and efficient approaches in low and middle frequency ranges. Here, the effectiveness of the technique used depends directly on the proper choice of the expansion functions and taking geometric particularities of the object into account.

The purpose of this presentation is to report a particular solution of the above problem for cylinders with arbitrarily polyhedral boundaries. It is based on the domain product technique (DPT). The DPT has been adapted to studying 2D multiangular structures in resonant frequency range. It regards domain of definition of the sought-for function as a common part (product) of some simple basic domains with separable geometry. The Mathieu function series are used to present fields in both the interior and exterior regions. In the framework of multiangular configurations, the DPT is not inferior to the FEM or the FDM in the sense of geometrical adaptability, but it is free of complications associated with presence of edges on the scatterer contour and with the mesh truncation for the exterior region. Over the past years, the DPT has been successfully used to solve interior [13-15] and exterior [16-24] problems of electromagnetics.

2. Scattering from conducting cylinders

2.1 Statement of the problem. The cross-section of the structure under consideration is shown in Fig. $1(a)$. Axis Oz of the basic coordinate system (x, y, z)

is directed along the generator of cylindrical boundaries. The time dependence $e^{i\omega t}$ is implied and suppressed throughout. E- or H-polarized wave with z-component $u_0(x, y)$ is incident on the perfectly conducting obstacles. In further discussion we shall distinguish between those cases as (E) and respectively (H).

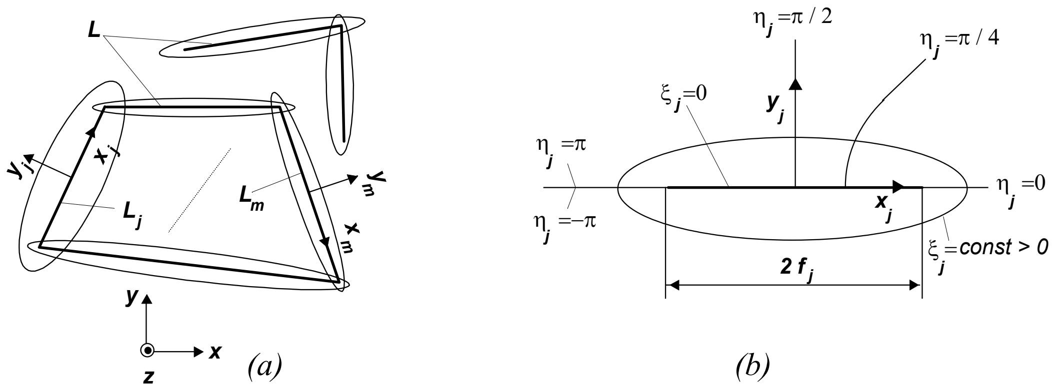


Fig. 1

Multiangular contour of the considered exterior region is a broken line L consisting of N segments: $L = \bigcup_{j=1}^{N} L_j$. For each the segment, a local Cartesian coordinate system (x_j, y_j) and an elliptic coordinate system (ξ_j, η_j) are introduced as

$$x_j = f_j \cosh\xi_j \cos\eta_j,\ y_j = f_j \sinh\xi_j \sin\eta_j,\ j=\overline{1,N} \quad (1)$$

Both systems are shown in Figs. $1(a,b)$. The origin of the system (x_j, y_j) is located at the center of the segment and axis Oy_j is directed into the domain of the field along the normal to L_j. The length of the jth segment is equal to $2f_j$.

Let us define the longitudinal component of the total field as

$$u = u_s + u_0 \quad (2)$$

where u_s is the scattering contribution due to the presence of the obstacles. The sought-for function u_s must satisfy the 2D Helmholtz equation with boundary conditions

$$u_s = -u_0,\ |x_j| < f_j,\ y_j = 0+,\ j=\overline{1,N}, \qquad \text{(E)} \quad (3)$$

$$\frac{\partial u_s}{\partial y_j} = -\frac{\partial u_0}{\partial y_j},\ |x_j| < f_j,\ y_j = 0+,\ j=\overline{1,N} \qquad \text{(H)} \quad (4)$$

If j refers to an open part of the boundary, condition (3) or (4) must be applied on both sides of the segment at $y_j = 0\pm$. In addition, the edge condition and the radiation condition, applied to the function u_s at infinity, must be satisfied.

2.2 Field representation. In elliptic coordinates, every segment of the boundary can be taken for a degenerate ellipse $\{(\xi_j, \eta_j) | \xi_j = 0; \eta_j \in (-\pi, \pi]\}$ and domain of the sought-for function u_s can be considered as a common part of simple infinite domains $\xi_j > 0$ $(j=\overline{1,N})$. Since Helmholtz's equation is a linear one, the quantity

u_s can be written in the form of superposition of some other functions, being solutions to the equation, as

$$u_s = \sum_{j=1}^{N} u_j \tag{5}$$

Suppose region $\xi_j > 0$ is the domain of definition of the function u_j, and this function satisfies the condition needed at infinity. In coordinates (ξ_j, η_j), region $\xi_j > 0$ has separable geometry and function u_j can be expressed as series of angular eigenfunctions. Assuming L_j is a segment of an open boundary contour and examining boundary conditions, we find out that u_j must have form

$$u_j = \sum_{n=1}^{\infty} D_n^j \, M_n(\xi_j, q_j) ma_n(\eta_j, q_j), \; j = \overline{1, N} \tag{6}$$

where

$$M_n(\xi_j, q_j) = \frac{Me_{n-1}^{(2)}(\xi_j, q_j)}{Me_{n-1}^{(2)}(0, q_j)}, \quad ma_n(\eta_j, q_j) = ce_{n-1}(\eta_j, q_j) \tag{7}$$

in the case of the Dirichlet boundary condition (3) [16], and

$$M_n(\xi_j, q_j) = \frac{Ne_n^{(2)}(\xi_j, q_j)}{Ne_n^{(2)}(0, q_j)}, \quad ma_n(\eta_j, q_j) = se_n(\eta_j, q_j) \tag{8}$$

in the case of the Neumann boundary condition (4) [13]. Here, $se_n(\eta_j, q_j)$ and $ce_n(\eta_j, q_j)$ are odd and even angular Mathieu functions, $Ne_n^{(2)}(\xi_j, q_j)$ and $Me_n^{(2)}(\xi_j, q_j)$ are relevant radial Mathieu functions [25], $q_j = (k_0 \, f_j \, / 2)^2, k_0 = 2\pi / \lambda$ and λ is the free-space wavelength. $\{D_n^j\}$ is a sequence of the unknown expansion coefficients. In fact, representations (5)-(8) coincide with those used in analysis of wave scattering from several separate strips [26].

Now, we assume that L_j is a part of a closed boundary contour. Then, function u_j is defined everywhere outside the L_j, whereas the boundary condition is prescribed only on one side of the segment turned to the region under consideration. The definition of this function will be unique provided the boundary condition is extended somewhat on the other side of the segment. Extending boundary values of the function u_j as an even function of variable η_j, we obtain representation (6), (7). The odd function extension results in representation (6), (8).

Examining expressions (5)-(8) [17], we can obtain

$$D_n^j = \frac{1}{\pi} \frac{Me_{n-1}^{(2)}(0, q_j)}{Me_{n-1}^{(2)\prime}(0, q_j)} \int_0^{\pi} \left[\frac{\partial u}{\partial y_j} \right] \sqrt{r_1^j r_2^j} \, ce_{n-1}(\eta_j, q_j) d\eta_j \tag{9}$$

when the even functions are used, and

$$D_n^j = \frac{1}{\pi} \int_0^{\pi} [u] se_n(\eta_j, q_j) d\eta_j \tag{10}$$

in the case of the odd function expansion. Here, $[\partial u / \partial y_j]$ and $[u]$ denote jumps of $\partial u / \partial y_j$ and u across L_j; r_1^j and r_2^j are distances between an observation point and end points of the segment; $M_m'(\xi_j, q_j)$ denotes the derivative of the function $M_m(\xi_j, q_j)$ with respect to ξ_j. Since u is the longitudinal component of the field, quantities $[u]$ and $[\partial u/\partial y_j]$ represent some (equivalent) magnetic and electric currents directed along the cross-sectional perimeter and z-directed respectively.

Suppose that in the vicinities of the ends of the segment, the z-directed current has singularities of the type $O(r^{-\gamma})$ being the same as for the x_j-directed component of the field. Then [17], $D_n^j = O(1/n^{2+\chi})$ as $n \to \infty$, where $\chi = 1 - 2\gamma$ and γ is the greatest of two possible powers differing from ½. The sequence $\{D_n^j\}$ approaches zero more rapidly with increase of n, if field singularities are absent or segment L_j is a separate strip. Odd function expansion is, in general, less beneficial in the sense of convergence because, unless $[u]$ vanishes at the ends of the segment, $D_n^j = O(1/n)$ only. The last condition holds provided L_j is cross-section of a separate strip (H) or series relating to both segments adjoining L_j have the even function form [24]. So, we assume further that in the (H) case all parts of the boundary contour are closed or consist of separate strips. Since $M_n(\xi_j, q_j) \sim \exp(-n\xi_j)$ (ξ_j fixed, $n \to \infty$), proper choice of the type of the expansion (6) ensures, with the above restriction, quite efficient representation of the field within the region considered and at its boundary.

2.3 Algebraization of the problem. Inserting (2) and (5)-(8) into (3) or (4) yields the infinite linear gebraic system

$$D_m^j + \sum_{p \neq j} \sum_{n=1}^{\infty} a_{mn}^{jp} D_n^p = c_m^j, \quad m = \overline{1, \infty}, \quad j = \overline{1, N} \qquad (11)$$

with

$$c_m^j = -\frac{2}{\pi} \int_0^{\pi} u_0 \big|_{\xi_j = 0} m a_m(\eta_j, q_j) d\eta_j \qquad \text{(E)} \qquad (12)$$

$$c_m^j = -\frac{2}{\pi M_m'(0, q_j)} \int_0^{\pi} \frac{\partial u_0}{\partial \xi_j} \bigg|_{\xi_j = 0} m a_m(\eta_j, q_j) d\eta_j \quad \text{(H)} \qquad (13)$$

The matrix elements take the forms

$$a_{mn}^{jp} = \frac{2}{\pi} \int_0^{\pi} [\ M_n(\xi_p, q_p) m a_n(\eta_p, q_p)\ ]_{\xi_j = 0} m a_m(\eta_j, q_j) d\eta_j \qquad \text{(E)} \qquad (14)$$

$$a_{mn}^{jp} = \frac{2}{\pi M_m'(0, q_j)} \int_0^{\pi} \frac{\partial}{\partial \xi_j} [M_n(\xi_p, q_p) m a_n(\eta_p, q_p)]_{\xi_j = 0} m a_m(\eta_j, q_j) d\eta_j \quad \text{(H)} \qquad (15)$$

It was shown in [17,18] that, dealing with those equations , one could apply truncation procedure provided the frequency of the incident field does not coincide with natural frequencies of the region supplementing domain of the field up to a complete plane. It should be noted that natural frequencies of the last region can be easily changed by placing an auxiliary boundary segment inside it (dashed line in Fig. 1(a)), adding an appropriate term into (5) and assigning a boundary condition on the segment introduced. Regarding the new segment as a perfectly conducting one, we shift natural frequencies along the real axis [18]. Prescribing auxiliary boundary condition of an impedance type, we make them complex valued.

3. Scattering from penetrable cylinders

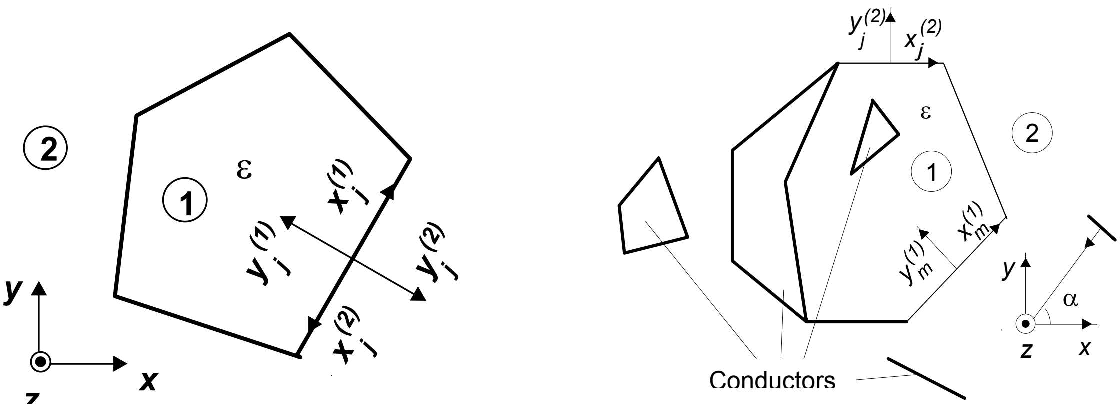
Consider, first, the problem of scattering from dielectric cylinder. Cross-section of the cylinder is shown in Fig.2(a). It is composed of regions 1 and 2. Region 1 is filled with a homogeneous and lossless material having relative permittivity and permeability ε and μ, respectively.

Two local Cartesian coordinate systems $\left(x_j^{(K)}, y_j^{(K)}\right)$ $(K=1,2)$ are introduced for each the segment $L_j\,(j=\overline{1,N})$. Axis $Oy_j^{(K)}$ of the Kth system is directed into the region of the same name. Orientation of the system is chosen in such a way that $\hat{x}_j^{(K)} \times \hat{y}_j^{(K)} = \hat{z}$. The region wave numbers are $k^{(1)} = (\varepsilon\mu)^{1/2} k_0$ and $k^{(2)} = k_0$.

Let us define the z-component of the total field in regions 1 end 2 as

$$u^{(1)} = u_s^{(1)} \tag{16}$$

$$u^{(2)} = u_s^{(2)} + u_0 \tag{17}$$



(a) Fig. 2 *(b)*

where $u_s^{(1)}$ and $u_s^{(2)}$ are scattering contributions in the interior and in the exterior. Instead of (3) and (4), the sought-for functions $u_s^{(1)}$ and $u_s^{(2)}$ must satisfy conditions, which guarantee continuities of the tangential electric and magnetic fields across interface

$$u^{(1)}\Big|_{y_j^{(1)}=0+} = u^{(2)}\Big|_{y_j^{(2)}=0+}, \quad |x_j^{(1)}| < f_j, \quad j=\overline{1,N} \tag{18}$$

$$\left.\frac{\partial u^{(1)}}{\partial y_j^{(1)}}\right|_{y_j^{(1)}=0+} = -P \left.\frac{\partial u^{(2)}}{\partial y_j^{(2)}}\right|_{y_j^{(2)}=0+}, \quad |x_j^{(1)}| < f_j, \quad j=\overline{1,N} \tag{19}$$

where $P=\mu$ (E) or $P=\varepsilon$ (H).

To apply the DPT, we again introduce local elliptic coordinate systems $(\xi_j^{(K)}, \eta_j^{(K)})$ as

$$x_j^{(K)} = f_j \cosh\xi_j^{(K)} \cos\eta_j^{(K)}, \; y_j^{(K)} = f_j \sinh\xi_j^{(K)} \sin\eta_j^{(K)}, \; j=\overline{1,N}, \; K=1,2 \tag{20}$$

That results in representation of the Kth region (it does not matter internal or external one) as a common part of the simple infinite domains $\xi_j^{(K)} > 0$, $j = \overline{1, N}$. In a manner similar to that used in the above section, we obtain

$$u_s^{(K)} = \sum_{j=1}^{N} u_j^{(K)} \tag{21}$$

$$u_j^{(K)} = \sum_{n=1}^{\infty} {}^K D_n^j\, M_n\left(\xi_j^{(K)}, q_j^{(K)}\right) ma_n\left(\eta_j^{(K)}, q_j^{(K)}\right),\ \ j = \overline{1, N},\ \ K = 1,2 \tag{22}$$

where $q_j^{(K)} = (k^{(K)} f_j / 2)^2$.

Substituting (16),(17) and (21),(22) into matching conditions (18),(19) yields an infinite algebraic system

$$ {}^1D_m^j + \sum_{p \neq j} \sum_{n=1}^{\infty} {}^{11}a_{mn}^{jp}\, {}^1D_n^p + \sum_{n=1}^{\infty} {}^{12}a_{mn}^{jj}\, {}^2D_n^j + \sum_{p \neq j} \sum_{n=1}^{\infty} {}^{12}a_{mn}^{jp}\, {}^2D_n^p = {}^1c_m^j \qquad m = \overline{1, \infty}, \quad j = \overline{1, N} \tag{23}$$

$$ \sum_{n=1}^{\infty} {}^{21}a_{mn}^{jj}\, {}^1D_n^j + \sum_{p \neq j} \sum_{n=1}^{\infty} {}^{21}a_{mn}^{jp}\, {}^1D_n^p + {}^2D_m^j + \sum_{p \neq j} \sum_{n=1}^{\infty} {}^{22}a_{mn}^{jp}\, {}^2D_n^p = {}^2c_m^j \qquad m = \overline{1, \infty}, \quad j = \overline{1, N} \tag{24}$$

Here, matrix elements ${}^{11}a_{mn}^{jp}$, ${}^{12}a_{mn}^{jp}$, ${}^{21}a_{nm}^{jp}$ and ${}^{22}a_{mn}^{jp}$ describe interaction among segments with different numbers belonging to one or different regions. They have the same type as integrals (14), (15) and are not cited here. Right-hand parts ${}^1c_m^j$ and ${}^2c_m^j$ have forms similar to (12) and (13). The other matrix elements, associated with interface, are as follows:

$$ {}^{12}a_{mn}^{jj} = (-1)^n I_{m-1,n-1} = (-1)^n \frac{2}{\pi} \int_0^{\pi} ce_{n-1}(\eta_j^{(1)}, q_j^{(2)}) ce_{m-1}(\eta_j^{(1)}, q_j^{(1)}) d\eta_j^{(1)} \tag{25}$$

$$ {}^{21}a_{mn}^{jj} = (-1)^{n+1} \frac{M_n'(0, q_j^{(1)})}{P M_m'(0, q_j^{(2)})} I_{n-1,m-1} \tag{26}$$

Since $ce_n(\eta, q) \sim \cos n\eta$, $n \to \infty$, functions $ce_n\left(\eta_j^{(1)}, q_j^{(2)}\right)$ and $ce_m\left(\eta_j^{(1)}, q_j^{(1)}\right)$ in (25) are asymptotically orthogonal. Based on this property, one can rewrite quantities ${}^{12}a_{mn}^{jj}$ and ${}^{21}a_{mn}^{jj}$ as

$$ {}^{12}a_{mn}^{jj} = (-1)^n \delta_{mn} + \alpha_{mn}^j, \quad {}^{21}a_{mn}^{jj} = (-1)^{n+1} \frac{1}{P} \delta_{mn} + \beta_{mn}^j \tag{27}$$

where α_{mn}^j and β_{mn}^j are some infinitesimal sequences. It can be shown that they approach zero, as $m, n \to \infty$, not more slowly than matrix elements ${}^{KL}a_{mn}^{jp}$ with $j \neq p$. It means, that on replacing ${}^1D_m^j$ and ${}^2D_m^j$ $(j = \overline{1, N})$ in (23) and (24) by new unknowns

$$ {}^1\widetilde{D}_m^j = {}^1D_m^j + (-1)^m\, {}^2D_m^j, \quad {}^2\widetilde{D}_m^j = {}^2D_m^j - \frac{(-1)^m}{P}\, {}^1D_m^j \tag{28}$$

we obtain a conventional DPT system, which can be solved using truncation procedure.

Representations (16),(17),(21) and (22) are also valid and in the case of the arbitrary penetrable cylinder, shown in Fig.2(b) and composed of dielectrics and conductors. Corresponding expansion coefficients can be found from the system obtained by combining matrix equations of the type (11)-(15) and (23)-(26) associated with conducting boundaries and interfaces, respectively.

4. Numerical examples

The purpose of this section is to present sample numerical results illustrating implementation of the above theory. Incident field is a plane wave

$$u_0(x,y) = u^+(x,y) = \exp[ik_0(x\cos\alpha + y\sin\alpha)] \qquad (29)$$

where α is the impinging angle relative to the x -axis.

First, for the sake of verification, we consider a triangular dielectric cylinder backed by a conducting strip. Note that in a particular case of $\varepsilon = 1$, solution to the problem should be the same as one obtained for the strip alone. The convergence of the algorithm versus truncation size M , being the order of the partial sums in (22) after truncation, is illustrated in Fig.3. As seen, quantity $|\partial u^{(1)}/\partial n|$ (E) and $|u^{(1)}|$ (H), which are proportional to magnitudes of the surface current densities, settle down to their final values for low enough truncation sizes. Therefore, they can be accurately evaluated using a few basis functions per λ-length of the boundary contour. In the case of $\varepsilon = 1$, solutions converge to the known values (dashed lines) obtained by the method of separation of the variables. Corresponding far-field pattern (not given here) level more rapidly with increasing order M . Roughly, truncation number $M_j = [l_j/(0.21\lambda)] + 2$ with $l_j = 2f_j\sqrt{\varepsilon} < 2\lambda$ is recommended for calculation of electrodynamic characteristics in far-zone. It is sufficient to plot quality graphs. This number can be reduced with increasing electrical length of the segment.

Fig.4 shows angular dependence of the normalized radar cross-section (RCS) for the 90° dihedral corner reflector with large enough electrical dimensions for the case of transverse magnetic excitation. A high-frequency limit of RCS [27] is given for comparison. During computation, cross-section of each face of the reflector was splitted into 10 segments. Results were obtained on an IBM-PC-compatible computer with Pentium-II CPU at 266 MHz. For 360 directions of incidence, the united running time was 32 s.

In Fig.5, to demonstrate flexibility of the technique in dealing with arbitrary geometry, we present the calculated angular dependence of RCS for a complex composite structure in the case of transverse electric excitation. That is a dielectric filled groove engraved in a ground plane. In the interior region of the groove, sought-for function $u^{(1)}$ is constructed in the form (16), (21) and (22) with $K = 1$. Exterior region 2 is a half-space. In the (H) case, function $u^{(2)}$ can be written as

$$u^{(2)} = u^+ + u^- + u_s^{(2)} \qquad (30)$$

where $u^-(x,y) = u^+(x,-y)$ is the known reflected plane wave. Function $u_s^{(2)}$ is a scattering contribution of the groove. It satisfies boundary conditions

$$\left.\frac{\partial u_s^{(2)}}{\partial \eta_1^{(2)}}\right|_{\eta_1^{(2)}=0} = \left.\frac{\partial u_s^{(2)}}{\partial \eta_1^{(2)}}\right|_{\eta_1^{(2)}=\pi} = 0 \qquad (31)$$

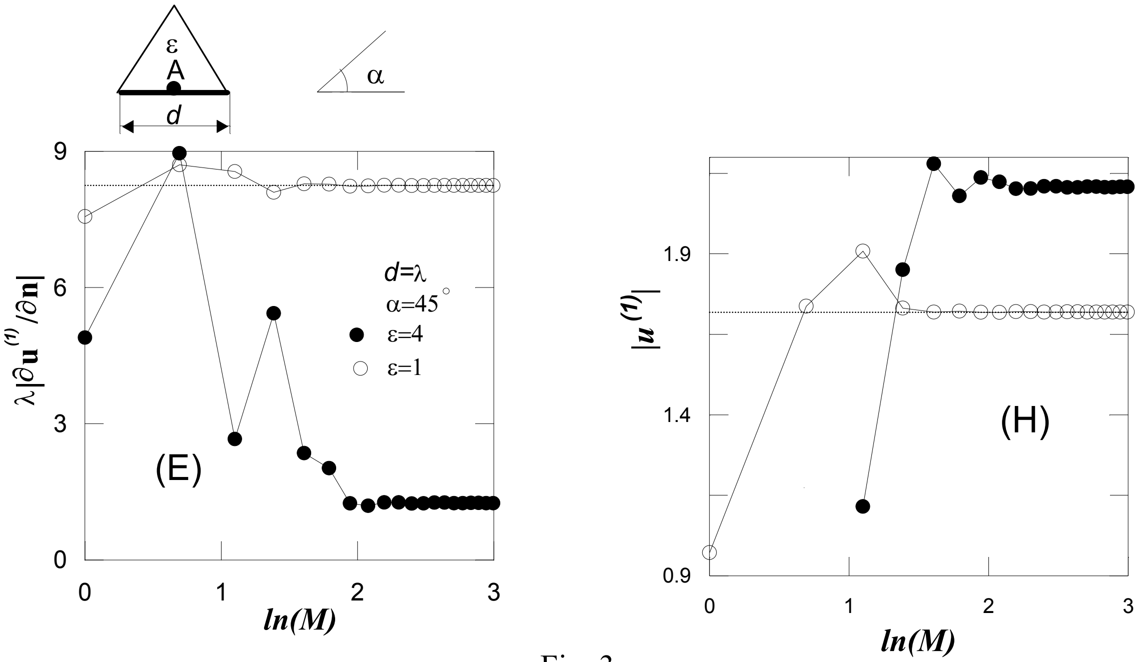


Fig. 3

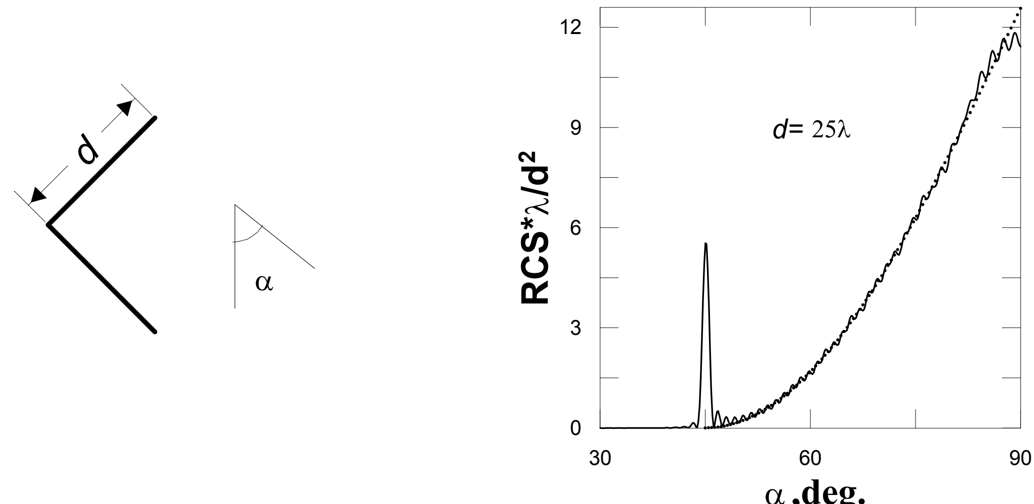


Fig. 4

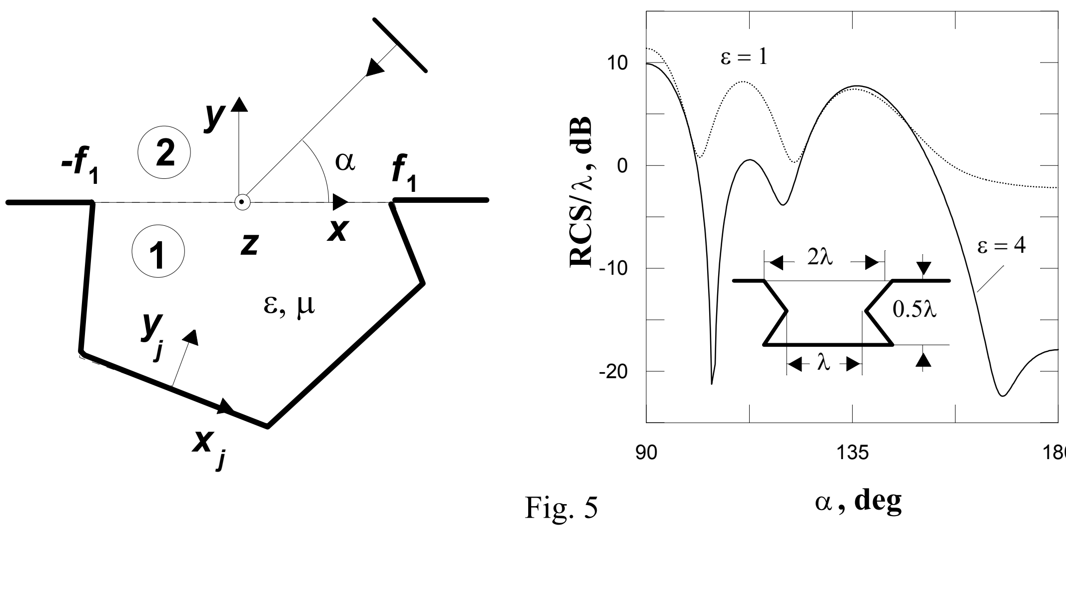


Fig. 5

and can be represented in the form (21) and (22), too, by setting $K=2$ and $N=1$. The only segment connected to region 2 is contour of the aperture.

Before closing this section, it should be noted that the technique could be applied directly and for the other type of excitation such, for example, as a line source.

5. Conclusion

Domain product technique formulation and solution to the problem of determining the field scattered by a two-dimensional system composed of multiple perfect conductors or (and) lossless dielectrics with polyhedral boundaries are reported. The system is excited either by E- or H-polarized primary wave. In both the interior and exterior subregions, the efficient field representations are attained in the forms of the Mathieu function expansions. Sample results are presented, which validate the theory and demonstrate its capability to treat complex configurations. Method is efficient in a wide enough frequency range and gives possibility of a substantial variation of the geometry of the structure without applying other mathematical models.

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